Flavour Physics from first-principles QCD computations: the case of kaon oscillation

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Graduiertenkolleg Mainz, 25.10.06

Motivation

- Determination of SM parameters + bounds on beyond the SM physics in LHC era requires precise control over hadronic effects.
- Obvious first-principles approach: lattice QCD.
- Many sources of systematic uncertainty to be brought under control:
  - Light dynamical quarks effects.
  - Control of symmetries / Renormalisation.
  - Cutoff dependences.
  - Conceptual issues.
- New era for lattice QCD — pave the way for precision studies by developing methods to control systematics.

Outline

- SM Flavourdynamics and kaon decay.
  - CKM matrix, CP violation, UT triangle and all that.
  - OPE and hadronic contributions to weak matrix elements.
  - Kaon decay and indirect CP violation.
- Lattice QCD.
  - Options and choices: conceptual and practical issues.
- A precise computation of B_K in quenched QCD.
  - tmQCD setup for weak matrix elements.
  - Anatomy of the computation and results.
- The (immediate) future: incorporating light dynamical quarks.

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SM flavour dynamics and Unitarity Triangle(s)

Non-trivial flavour dynamics of the SM in quark sector encoded in the CKM matrix.

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} W_\mu^+ (\bar{u}_L, \bar{c}_L, \bar{t}_L) \gamma_\mu V_{\rm CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + \text{h.c.}$$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Three mixing angles, one CP-violating phase.

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$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + \mathrm{O}(\lambda^4)$$

$$\bar{\rho} = \rho\left(1-\frac{\lambda}{2}\right), \quad \bar{\eta} = \eta\left(1-\frac{\lambda}{2}\right)$$

Three mixing angles, one CP-violating phase.

$$\lambda = 0.2258(14)\,, \quad A = 0.82(1)$$

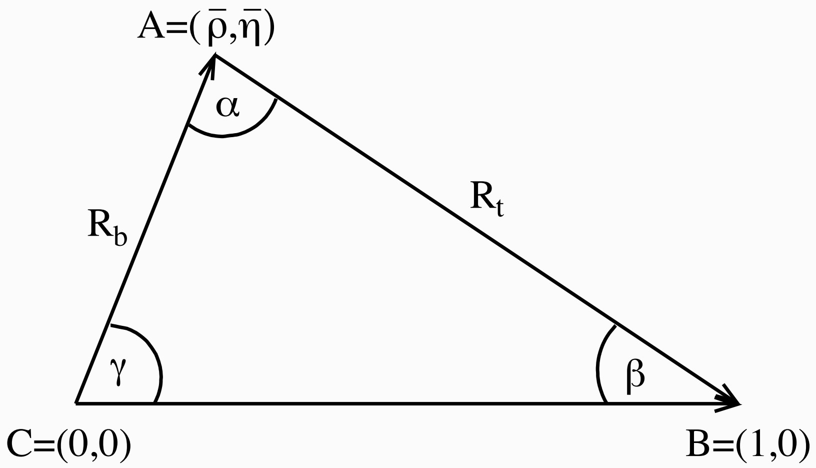
A and λ determined from tree-level decays of K and B mesons.

SM flavour dynamics and Unitarity Triangle(s)

$V^\dagger_{\rm CKM} V_{\rm CKM} = \mathbf{1} \quad \Rightarrow$ 9 constraints on CKM parameters / 6 triangle relations

Only two triangles have all sides of size λ^3:

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$
$$V_{ud}V^*_{td} + V_{us}V^*_{ts} + V_{ub}V^*_{tb} = 0$$

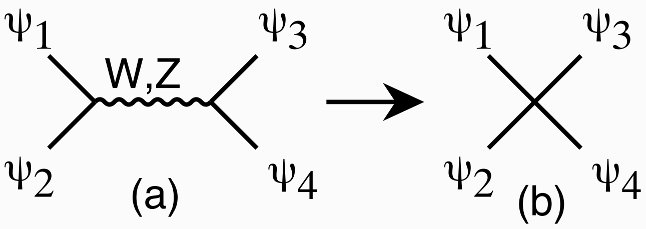


$$\overline{AB} = \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} \qquad \alpha = \arg\left(-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right)$$
$$\overline{BC} = 1 \qquad \beta = \arg\left(-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right)$$
$$\overline{CA} = \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} \qquad \gamma = \arg\left(-\frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}}\right)$$

CP violation proportional to the Jarlskog invariant $J = {\rm Im}\left\{V_{ij}V_{kl}V^*_{il}V^*_{kj}\right\}$

Effective theory for weak interactions and UT analysis

High- and low-energy scales separated via Operator Product Expansion:



$$g_{\mathrm{w}}^2 \longrightarrow \frac{g_{\mathrm{w}}^2}{8M_W^2} \simeq \frac{G_{\mathrm{F}}}{\sqrt{2}}$$

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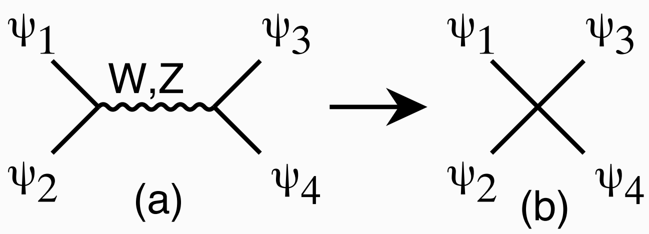

$$g_{\rm w}^2 \quad \longrightarrow \quad \frac{g_{\rm w}^2}{8M_W^2} \simeq \frac{G_{\rm F}}{\sqrt{2}}$$

$$\mathcal{A}(i \to f) \approx \langle f | H_{\rm W}^{\rm eff} | i \rangle$$

$$H_{\rm W}^{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \sum_k f_k(V_{\rm CKM}) C_k(\mu/M_{\rm W}) \bar{O}_k(\mu)$$

Effective theory for weak interactions and UT analysis

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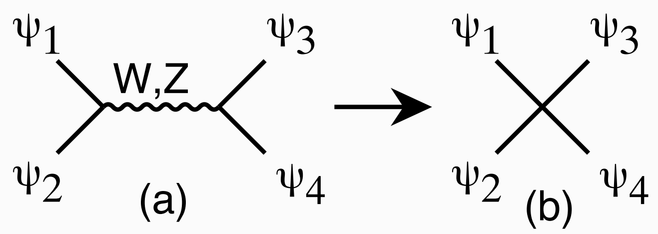

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CKM parameters

$$H_{\rm W}^{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \sum_k f_k(V_{\rm CKM}) C_k(\mu/M_W) \bar{O}_k(\mu)$$

Wilson coefficients — high energy, NLO computation

Composite operators — low energy (hadronic) scales

Effective theory for weak interactions and UT analysis

Insertion of

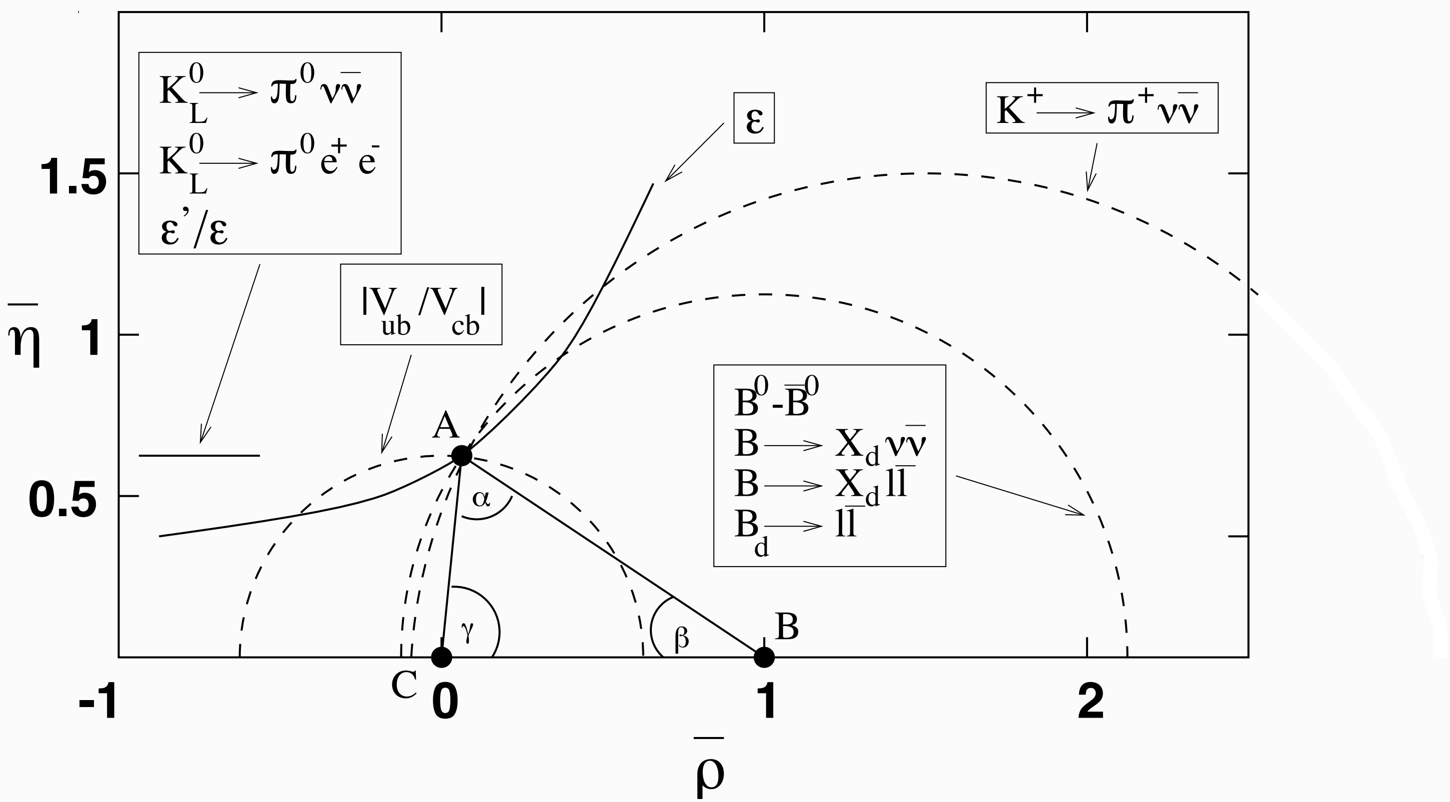
- experimental values of decay amplitudes,
- measured values for *A* and λ,
- computed values of Wilson coefficients and hadronic matrix elements

yields a geometric locus on the $(\bar{\rho}, \bar{\eta})$ plane.

$$\mathcal{A}(i \to f) \approx \langle f | H_{\rm W}^{\rm eff} | i \rangle$$

$$H_{\rm W}^{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} \sum_k f_k(V_{\rm CKM}) C_k(\mu / M_W) \bar{O}_k(\mu)$$

Effective theory for weak interactions and UT analysis



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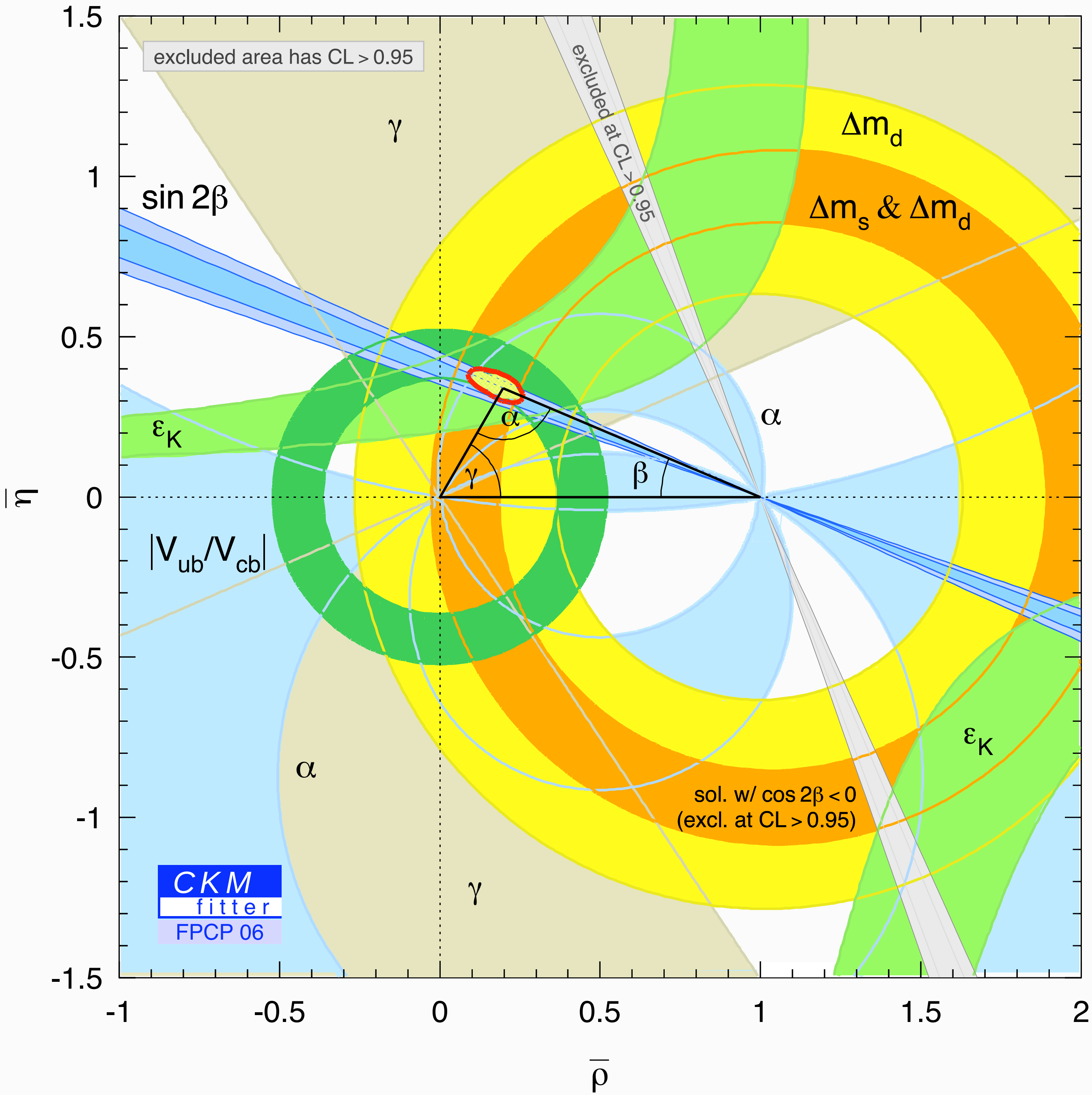


Kaon decay and CP violation

Kaons = lightest mesons with strangeness.

$$K^+ \sim u\bar{s}\,, \quad K^0 \sim d\bar{s}\,, \quad \bar{K}^0 \sim \bar{d}s\,, \quad K^- \sim \bar{u}s$$

$$m_{K^\pm} = 493.677 \pm 0.016 \text{ MeV}$$

$$(\text{cf } m_{\pi^\pm} \simeq 140 \text{ MeV})$$

$$m_{K^0} = 497.648 \pm 0.022 \text{ MeV}$$

K^0–$\bar{K}^0$ (CP eigenstates) mix into mass eigenstates due to CP violating electroweak effects:

K_S^0 predominant decay modes (99.9%) $K_S^0 \to \pi^+\pi^-\,, \quad K_S^0 \to \pi^0\pi^0$

K_L^0 predominant decay modes (99.8%) $K_L^0 \to \pi e \nu_e\,, \; K_L^0 \to \pi\mu\nu_\mu\,, \; K_L^0 \to 3\pi$

Kaon decay and CP violation

CP violation standard observables:

$$A_L = \frac{\Gamma(K_L^0 \to \pi^- \ell^+ \nu_\ell) - \Gamma(K_L^0 \to \pi^+ \ell^- \nu_\ell)}{\Gamma(K_L^0 \to \pi^- \ell^+ \nu_\ell) + \Gamma(K_L^0 \to \pi^+ \ell^- \nu_\ell)}$$

$$A_L = (3.32 \pm 0.06) \times 10^{-3}$$

$$\eta_{+-} = \frac{A(K_L^0 \to \pi^+ \pi^-)}{A(K_S^0 \to \pi^+ \pi^-)} \equiv \epsilon + \epsilon'$$

$$|\epsilon| = (2.232 \pm 0.007) \times 10^{-3}$$

$$\phi_\epsilon = (43.5 \pm 0.7)^\circ$$

$$\eta_{00} = \frac{A(K_L^0 \to \pi^0 \pi^0)}{A(K_S^0 \to \pi^0 \pi^0)} \equiv \epsilon - 2\epsilon'$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \epsilon'/\epsilon = (1.66 \pm 0.26) \times 10^{-3}$$

CP violation in mixing:

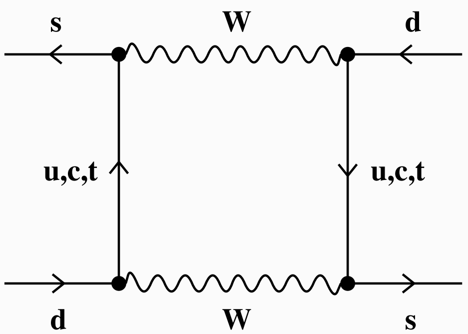
$$|K_S^0\rangle = p|K^0\rangle + q|\bar{K}^0\rangle$$

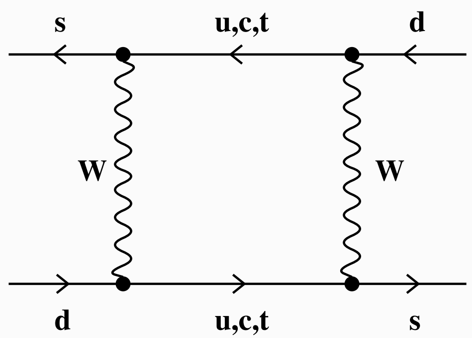
$$|K_L^0\rangle = p|K^0\rangle - q|\bar{K}^0\rangle$$

$$\frac{p}{q} = \frac{1+\tilde{\epsilon}}{1-\tilde{\epsilon}}$$

Wu-Yang phase convention: $\tilde{\epsilon} = \epsilon$

Kaon decay and CP violation





$$O_{\mathrm{LL}}^{\Delta S=2} = [\bar{s}\gamma_\mu(\mathbf{1}-\gamma_5)d][\bar{s}\gamma_\mu(\mathbf{1}-\gamma_5)d]$$

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1\, S_0(x_c) - \eta_3\, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2\, S_0(x_t)]\}$$

$$C_\epsilon = \frac{G_{\mathrm{F}}^2 F_K^2 m_K M_W^2}{6\sqrt{2}\pi^2 \Delta m_K} = 3.837 \times 10^4$$

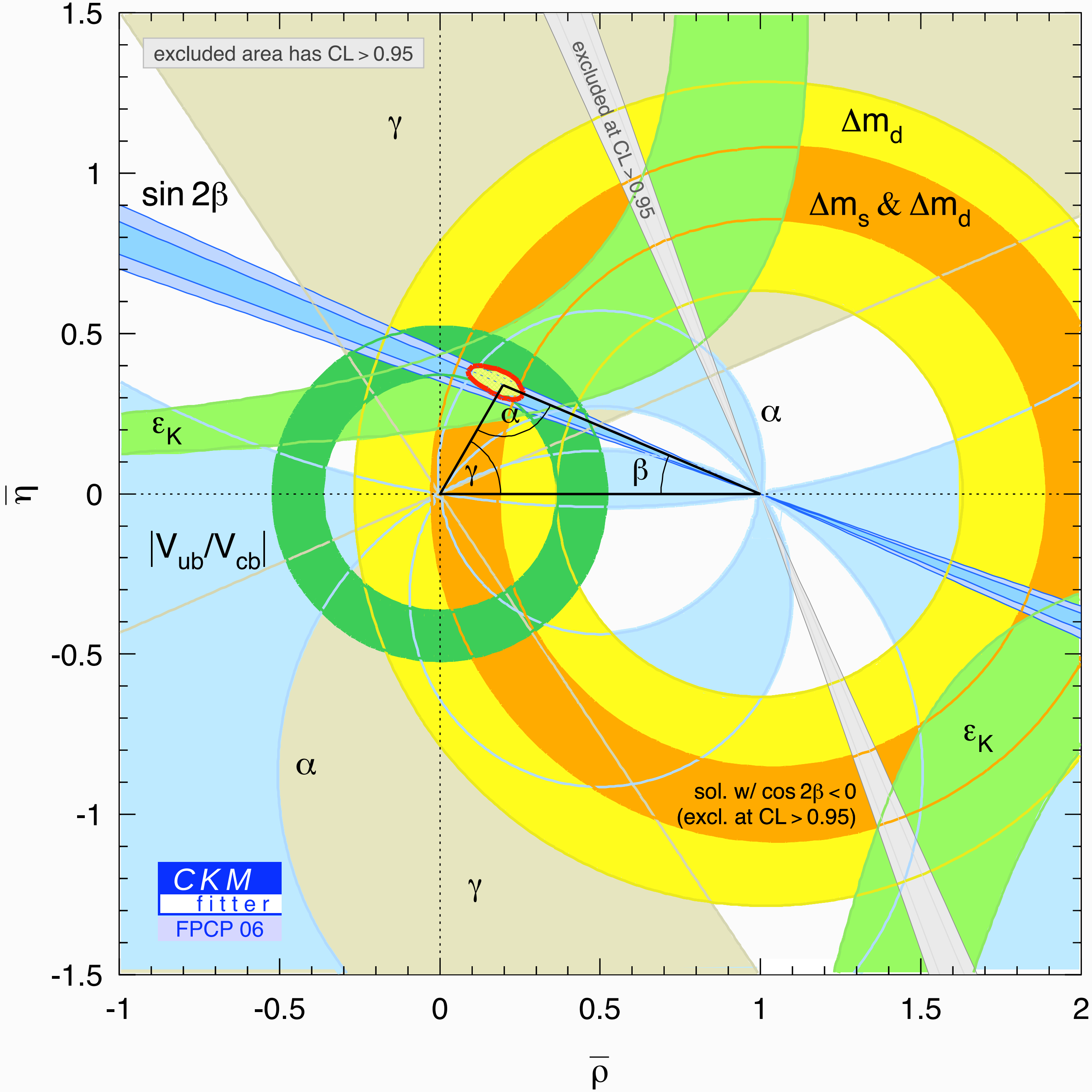
$$\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}$$

η_k , S_0 → short distance effects

Put in NLO PT + Cabibbo angle + A + $m_{c,t}$:

$$\bar{\eta}(1.4 - \bar{\rho})\, \hat{B}_K \approx 0.40$$

Kaon decay and CP violation



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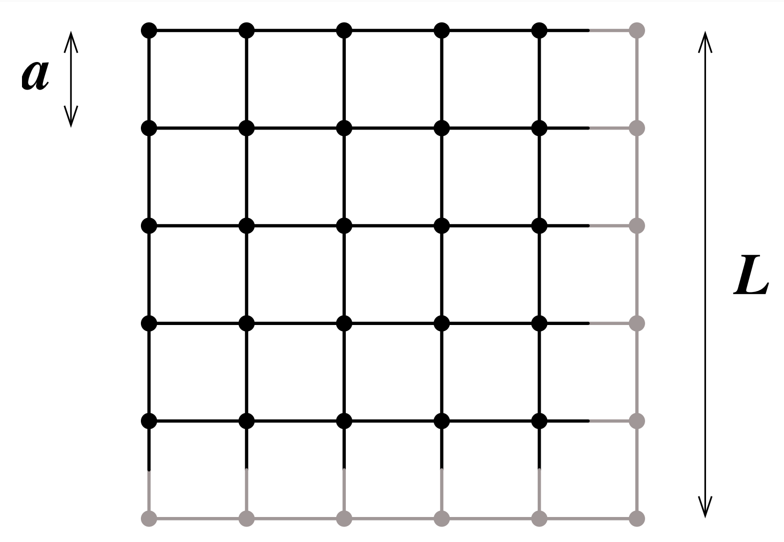
Lattice QCD

- Lattice spacing and finite volume effects (UV and IR cutoffs).
- Light quark masses difficult to simulate (hard lower bounds for some regularisations).
- Good field-theoretical control essential.



Simulate wide range of values of a, L, m to control all extrapolations.

$$a \lesssim 0.05 \text{ fm} \ \rightarrow \ 0.1 \text{ fm}$$

$$m_\pi : \ 500 \text{ MeV} \ \rightarrow \ \ldots$$

$$L \geq 2 \text{ fm} , \quad m_\pi L \geq 3$$

Lattice QCD — which one?

LQCD formulation not unique.

$$S_{\text{lat}} = S_0 + aS_1 + a^2 S_2 + \ldots$$

$$\mathcal{O}_{\text{lat}} = \mathcal{O}_0 + a\mathcal{O}_1 + a^2\mathcal{O}_2 + \ldots$$

Emphasis on different requirements:

- Conceptual clarity — avoid tradeoffs on basic properties.
- Preserve symmetries (or break them in a controlled way).
- Reduce lattice spacing effects.
- Affordable numerical performance.

Lattice QCD — which one?

LQCD formulation not unique.

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$$\mathcal{O}_{\rm lat} = \mathcal{O}_0 + a\mathcal{O}_1 + a^2\mathcal{O}_2 + \ldots$$

Classic example: chiral symmetry.

- *Wilson-type fermions*: chiral symmetry broken.

$$\langle [\partial_\mu A^i_\mu(x) - 2mP^i(x)]\Phi_1(y_1)\cdots\Phi_n(y_n)\rangle = \text{contact terms} + \mathrm{O}(a^n)$$

- *Ginsparg-Wilson fermions*: chiral symmetry exact with

$$\hat{\gamma}_5 = \gamma_5(\mathbf{1} - aD)\,, \quad \gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

Lattice QCD — which one?

No compromise on good field-theoretical control:

- Wilson fermions.
  - Plain — large cutoff effects.
  - $O(a)$ improved — complicate renormalisation problems.
  - Twisted mass QCD — trade axial/vector symmetries, simplify renormalisation.
- *Exact* Ginsparg-Wilson fermions (overlap): numerically very demanding, some systematic uncertainties difficult to control.

Choice for this work: tmQCD.

B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987, 1998; Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} \; - \; \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)]}_{O_{\mathrm{VA+AV}}}$$

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$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4}\Delta_k(g_0^2)O_k(a)\right]$$

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Vanishes if chiral symmetry is preserved (at least partially)

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$$\bar{O}_{\mathrm{VA+AV}} = \lim_{a\to 0} Z_{\mathrm{VA+AV}}(g_0^2, a\mu)\, O_{\mathrm{VA+AV}}(a)$$

Protected from mixing by discrete symmetries

Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly exactly.

- Wilson 1: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV}).

  Becirevic et al. 2000

- Wilson 2: tmQCD (different symmetry breaking pattern, 3-point function with O_{VV+AA}).

  ALPHA, Frezzotti, Grassi, Sint & Weisz, 2001

  ALPHA, Dimopoulos et al. 2004

  ALPHA, Dimopoulos et al. 2006

- tmQCD bonus: push safely towards low quark masses in quenched simulations.

Twistəd mass QCD

Break flavour symmetry in non-trivial direction in flavour space → preserve different subgroup.
No free lunch: break P,T, vector symmetries.

- Originally (re)proposed to avoid exceptional configurations in quenched computations.

  Frezzotti, Grassi, Sint, Weisz 2001

- Control of chiral symmetry breaking allows for simpler renormalisation properties → "mimic" exact chiral symmetry.

  Frezzotti, Grassi, Sint, Weisz 2001

  CP, Sint, Vladikas 2004

  Frezzotti, Rossi, 2004

- Interest outburst after Frezzotti and Rossi's argument for automatic O(a) improvement.

  Frezzotti, Rossi 2004

Twistǝd mass QCD

Basic setup: two mass-degenerate light flavours.

$$D_{\rm tmQCD} = \frac{1}{2}\gamma_\mu(\nabla^*_\mu + \nabla_\mu) + m_0 + e^{-i\alpha\gamma_5\tau_3}\left(-\frac{ar}{2}\nabla^*_\mu\nabla_\mu + m_{\rm cr} + \frac{i}{4}c_{\rm sw}\sigma_{\mu\nu}\hat{F}_{\mu\nu}\right)$$

- Facultative Sheikholeslami-Wohlert term to remove O(a) effects.
- α=0 → standard Wilson fermions.
  - Additional degree of freedom used to control chiral symmetry breaking.
  - Additive mass renormalisation preserved wrt standard case.
- Precise knowledge of improvement coeffiecients and renormalisation factors inherited from previous studies.

ALPHA non-perturbative renormalisation programme

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ALPHA Collaboration quenched computation of B_K

Guagnelli, Heitger, CP, Sint, Vladikas JHEP 03 (2006) 088

Palombi, CP, Sint JHEP 03 (2006) 089

Dimopoulos, Heitger, Palombi, CP, Sint, Vladikas NPB 749 (2006) 69

- tmQCD → no operator mixing, no exceptional configurations.
- SF non-perturbative renormalisation.
- Various physical volumes: check control of finite volume effects.
- Two different regularisations: check control of the continuum limit.
- N.B.: action is O(a) improved, but four-fermion operator is *not* ⇒ continuum limit approached linearly in a.
- Computations performed on the APEMille installation @ DESY-Zeuthen.

The case for a precise quenched Wilson computation of B_K

- Minimal conceptual uncertainties (cf. staggered, DW fermions).

- Numerically cheap ⇒ control cutoff dependence (cf. overlap fermions).

- Mature non-perturbative renormalisation techniques (cf. all other regularisations).

- Control/understanding of all quenched systematics essential to set up techniques and set target precision in unquenched computation.

tmQCD regularisations for B_K

<u>*π/2* strategy</u>:

→ fully ($\alpha=\pi/2$) twisted *u*-*d* doublet

→ untwisted *s* quark

<u>*π/4* strategy</u> (specially devised for quenched case):

→ ($\pi/4$) twisted *s*-*d* doublet

→ other flavours untwisted

in both cases $O_{\rm VV+AA}$ renormalises multiplicatively

Quenched simulations

- $\pi/2$:

  $4 \times \beta,\ a \sim 0.06 - 0.09\,\mathrm{fm},\ L \sim 1.4 - 1.9\,\mathrm{fm},\ T/L \sim 2.3 - 3.0,\ m_{\mathrm{PS}} \sim 640 - 830\,\mathrm{MeV}$

- $\pi/4$:

  $5 \times \beta,\ a \sim 0.05 - 0.09\,\mathrm{fm},\ L \sim 1.9 - 2.2\,\mathrm{fm},\ T/L \sim 2.0 - 2.6,\ m_{\mathrm{PS}} \sim 460 - 540\,\mathrm{MeV}$

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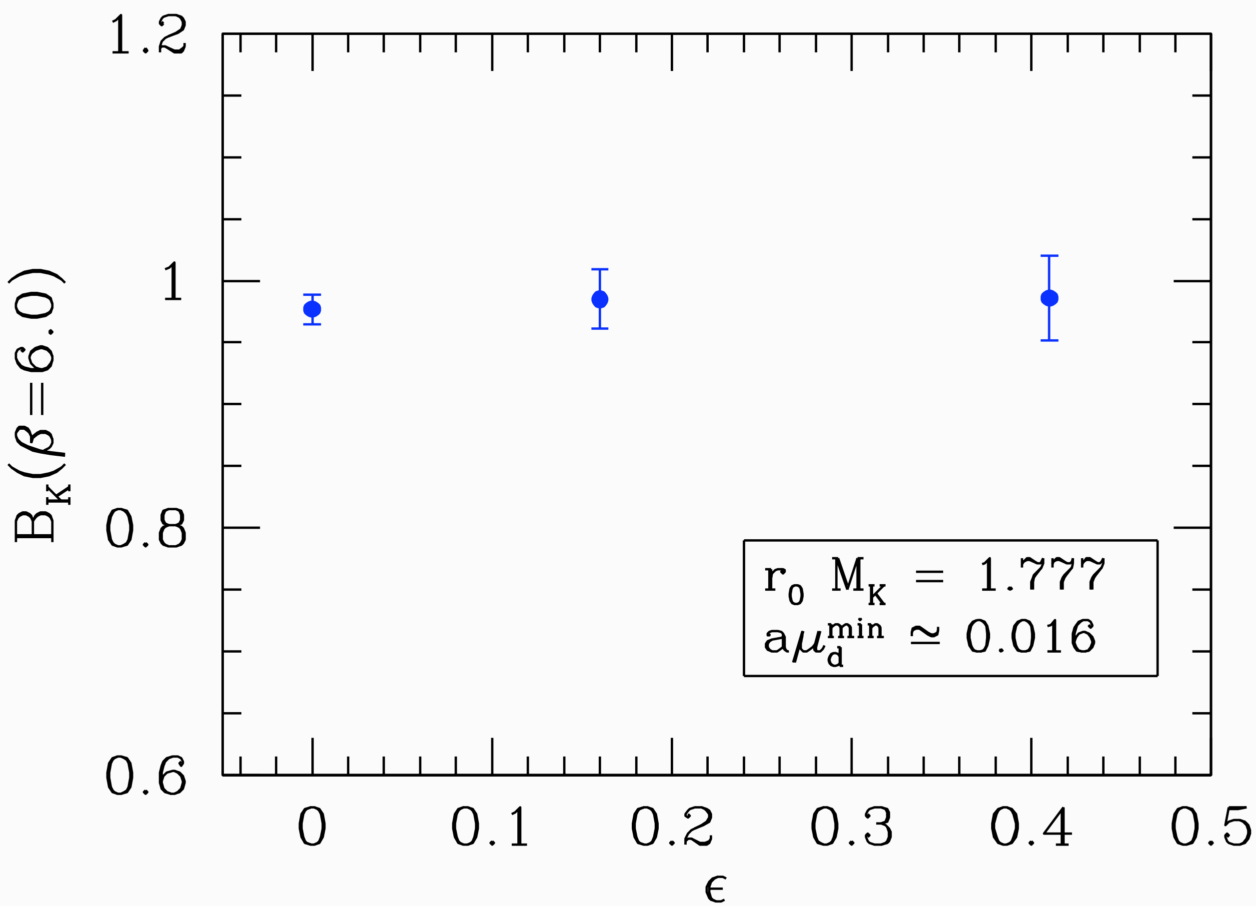
- $m_s = m_d$ limit. Physical SU(3) breaking effects checked to be small up to moderate values for the strange-down splitting.

Physical SU(3) breaking effects



$$\epsilon = \frac{M_s - M_d}{M_s + M_d}$$

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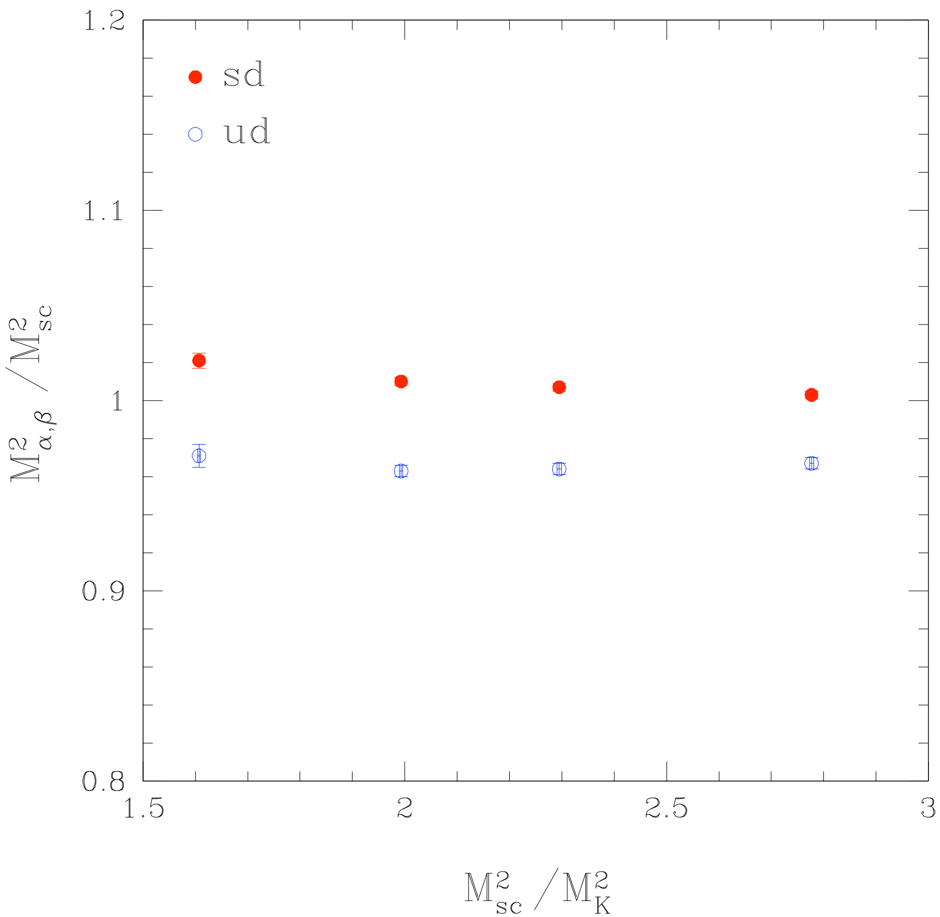
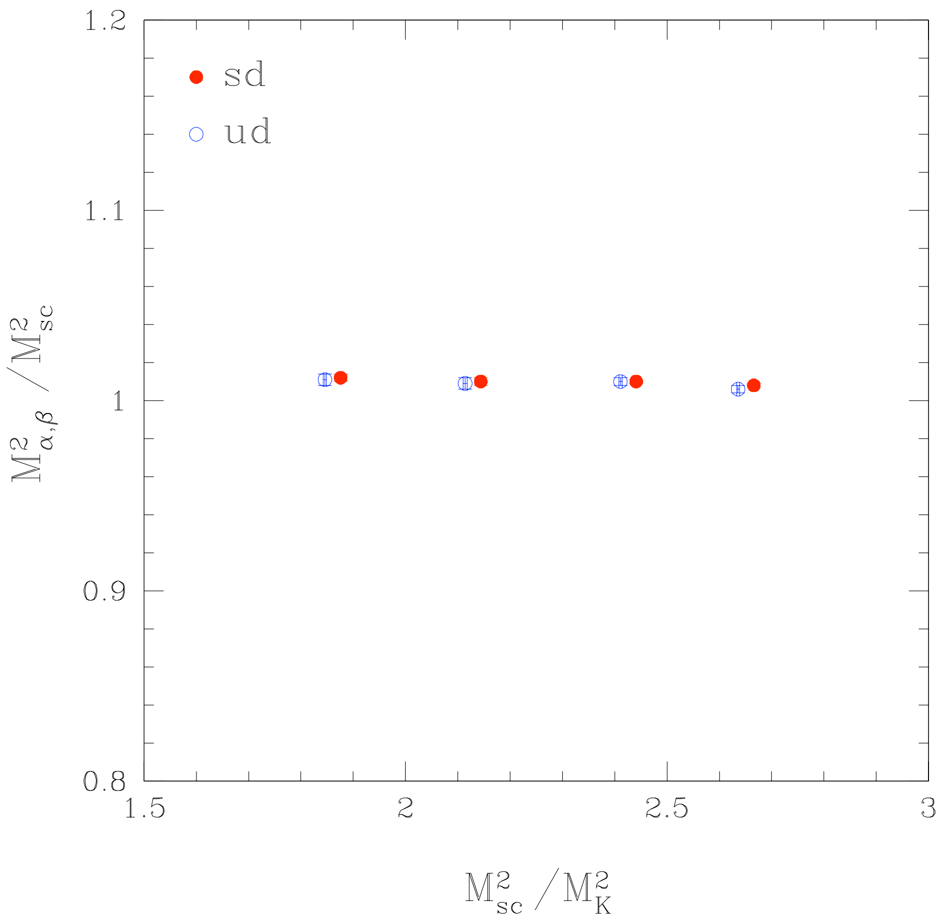
- Spurious $SU(3)_V$ breaking due to twist checked to be of order few %, converges to 0 in the continuum limit.

Twisted mass-induced vector symmetry breaking



$\beta = 6.0$



$\beta = 6.3$

Quenched simulations

- $\pi/2$:

  $4\times\beta,\ a\sim 0.06-0.09\,\mathrm{fm},\ L\sim 1.4-1.9\,\mathrm{fm},\ T/L\sim 2.3-3.0,\ m_{\mathrm{PS}}\sim 640-830\,\mathrm{MeV}$

- $\pi/4$:

  $5\times\beta,\ a\sim 0.05-0.09\,\mathrm{fm},\ L\sim 1.9-2.2\,\mathrm{fm},\ T/L\sim 2.0-2.6,\ m_{\mathrm{PS}}\sim 460-540\,\mathrm{MeV}$

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- Approach to continuum limit remains delicate.

Approach to continuum: non-perturbative renormalisation

ALPHA, JHEP 03 (2006) 088 & 089

- SF technique via finite size scaling: split renormalisation into
  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
  - NP running to very high scales (~100 GeV) where contact with PT is made.

$$\hat{B}_K = (\alpha_s(\mu))^{-\gamma_0/2b_0} \exp\left\{ - \int_0^{\bar{g}(\mu)} \mathrm{d}g \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g} \right] \right\} \left[\lim_{a\to 0} Z(g_0^2, a\mu)\, B_K(a) \right]$$

Approach to continuum: non-perturbative renormalisation

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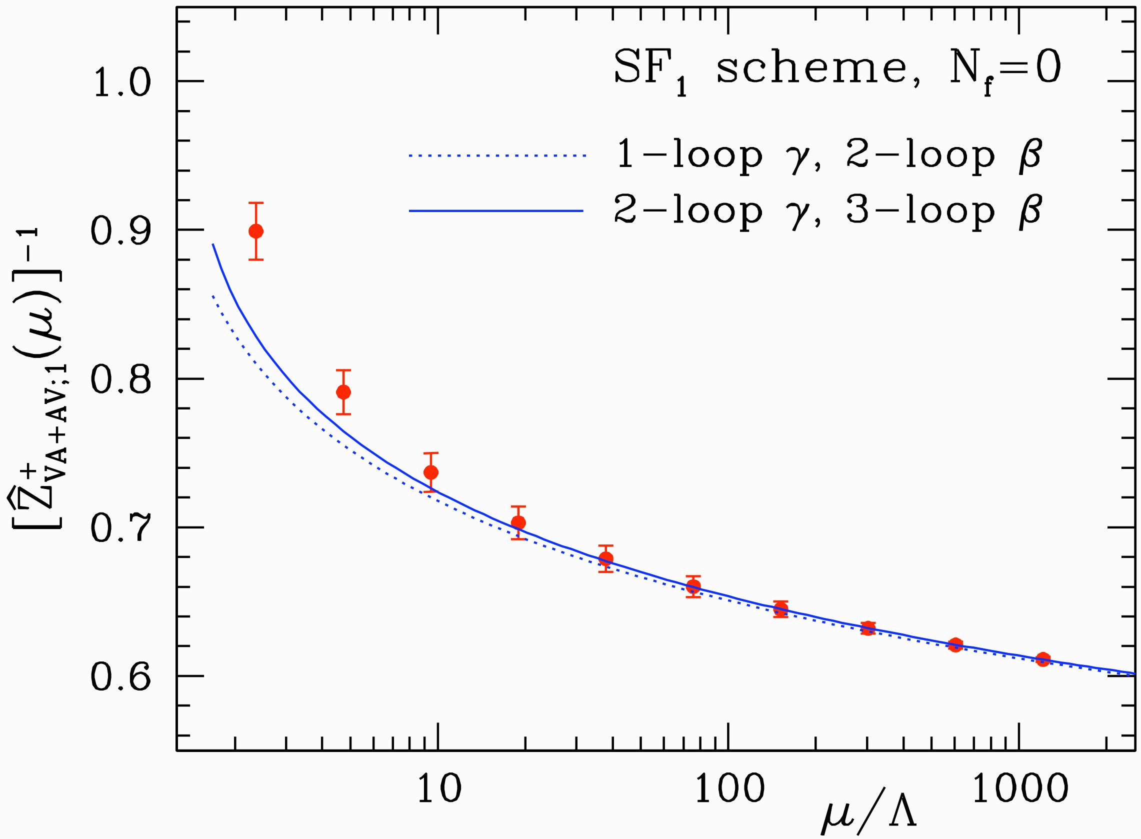

Continuum limit

- Combined linear extrapolation of the two regularisations, using ALPHA determination of current normalisations and improvement coefficients.
- Criteria:
  - Discard points on which the impact of $O(a^2)$ ambiguities are checked to be beyond the 1 sigma level.
  - Discard points for which (impossible to fit) curvature in a dependence is manifest.

Continuum limit

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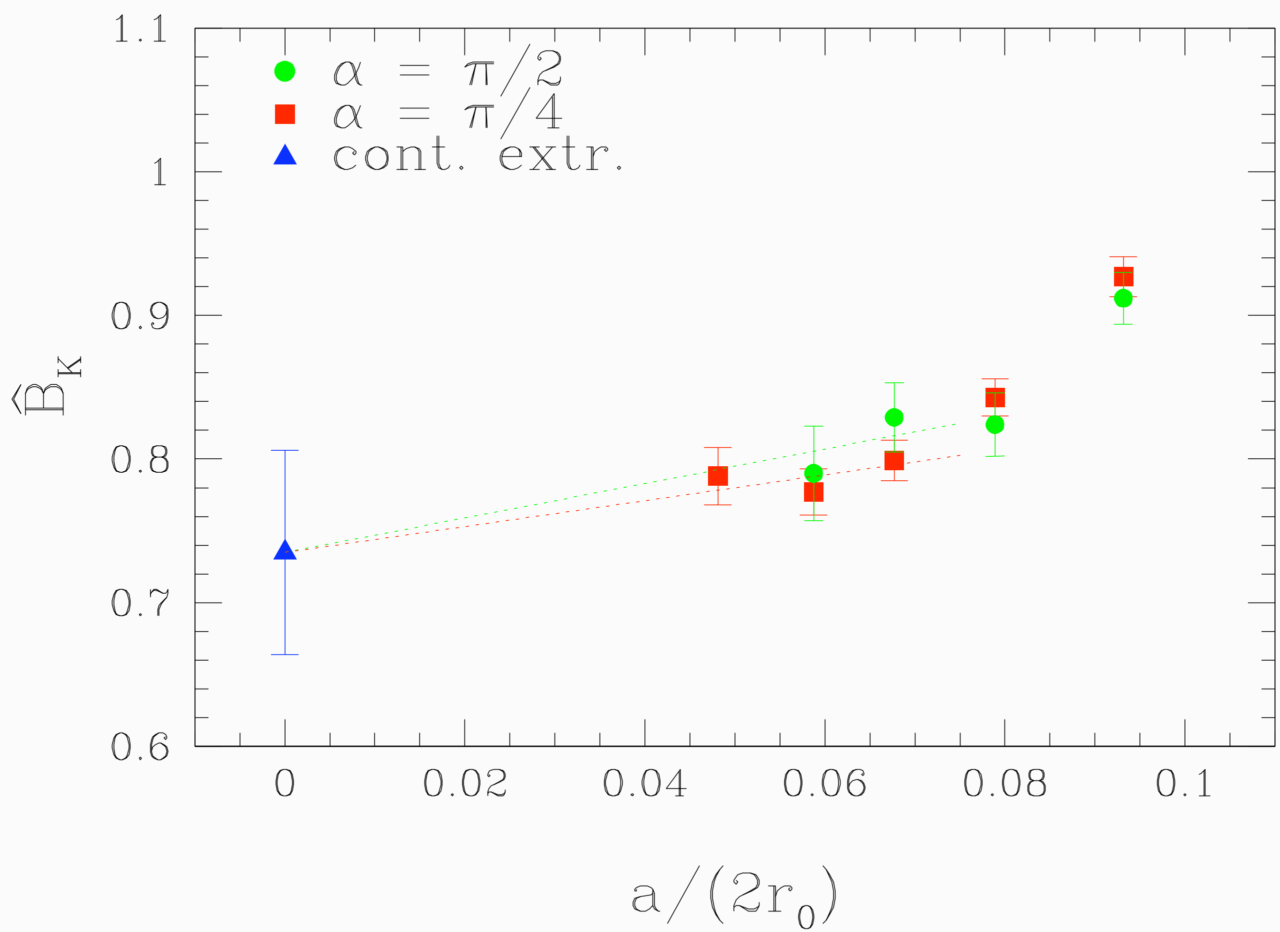


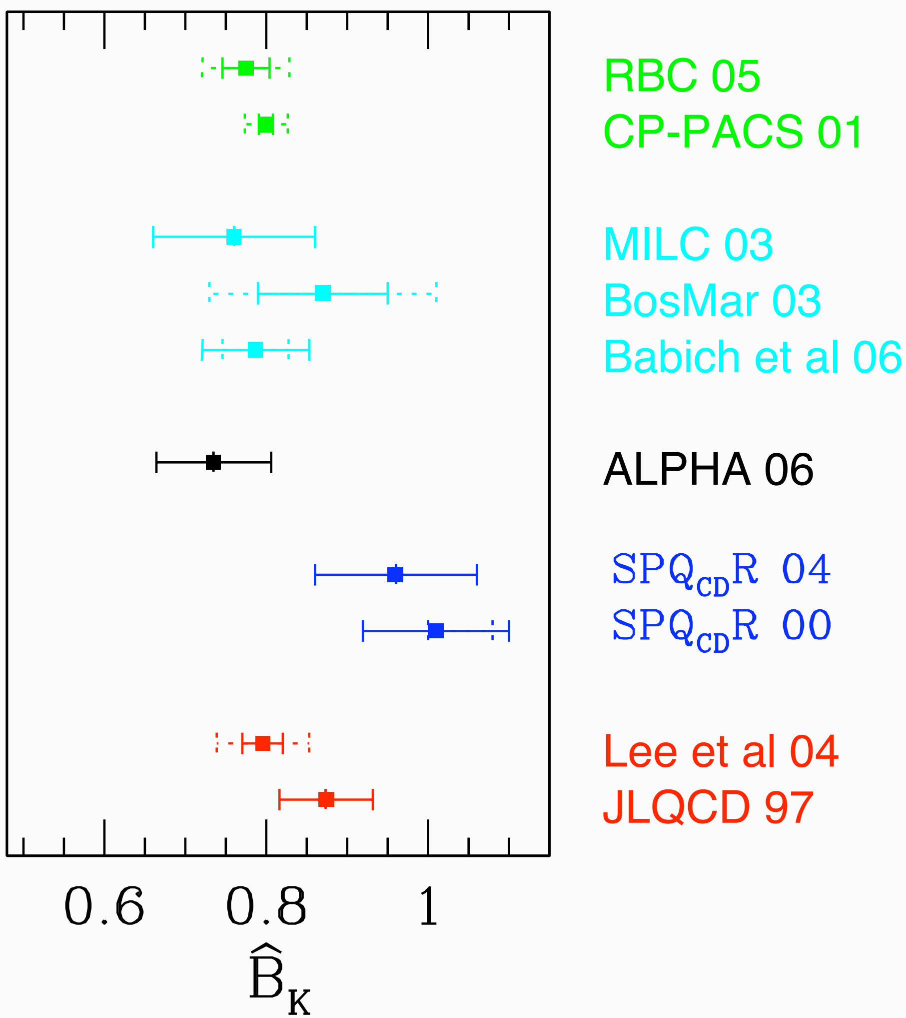
$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.534(52)$$

Dimopoulos et al., in preparation

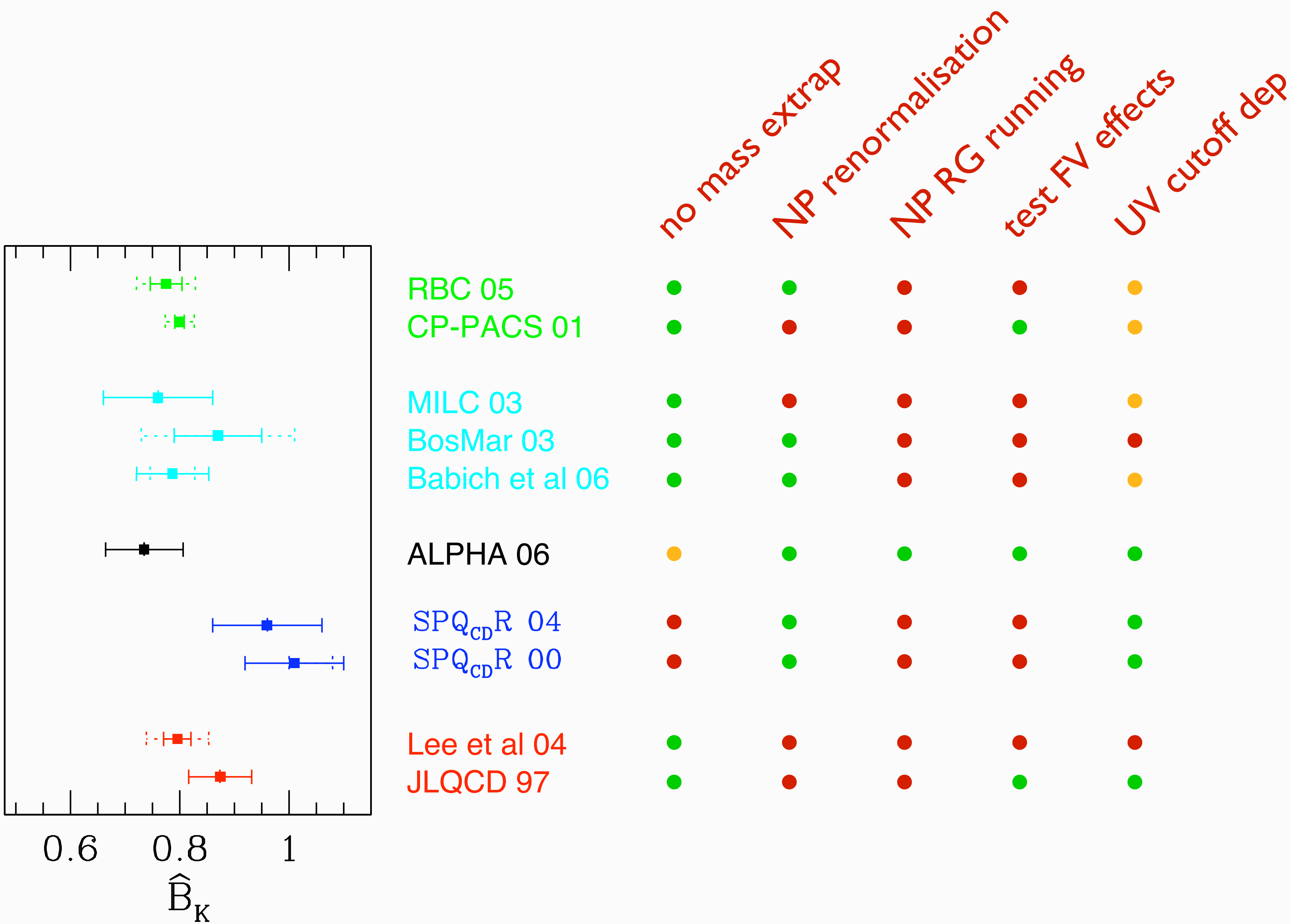
Crucial for the future: adopt framework that keeps cutoff effects at $O(a^2)$.

Comparison with quenched literature



Difference with other Wilson fermion computations mainly due to method employed to extract B_K.

Comparison with quenched literature



Outline

- SM Flavourdynamics and kaon decay.
  - CKM matrix, CP violation, UT triangle and all that.
  - OPE and hadronic contributions to weak matrix elements.
  - Kaon decay and indirect CP violation.
- Lattice QCD.
  - Options and choices: conceptual and practical issues.
- A precise computation of B_K in quenched QCD.
  - tmQCD setup for weak matrix elements.
  - Anatomy of the computation and results.
- The (immediate) future: incorporating light dynamical quarks.

Entering a new era: dynamical light quarks

Pessimistic early 00's views on light quarks defused after 5 years.

$$5\left[\frac{20\ \mathrm{MeV}}{m}\right]^{3}\left[\frac{L}{3\ \mathrm{fm}}\right]^{5}\left[\frac{0.1\ \mathrm{fm}}{a}\right]^{7}$$

Ukawa, Berlin Lattice Conference (2001)

$$0.05\left[\frac{20\ \mathrm{MeV}}{m}\right]^{1}\left[\frac{L}{3\ \mathrm{fm}}\right]^{5}\left[\frac{0.1\ \mathrm{fm}}{a}\right]^{6}$$

Giusti, Tucson Lattice Conference (2006)

Tflops-years required for 100 gauge fields in two-flavour QCD (O(a) improved Wilson quarks, T=2L)

- Better physical understanding of algorithmic issues → breakthrough in algorithmic efficiency. Many groups going light with Wilson fermions.

ALPHA, CERN-ToV, ETMC, PACS-CS, QCDSF ...

- Moore's law helps, too ... Very powerful installations allow for dynamical chiral fermions.

JLQCD 2006

Entering a new era: dynamical light quarks

Public databases of dynamical configurations / public code available.

- MILC configurations/code (but: beware of staggered fermions!).

  Sharpe, Lattice 2006

- International Lattice Data Grid.

- M. Lüscher's highly optimised PC cluster code released under GPL.

- No foreseeable difficulty to extend to 2+1(+1) dynamical flavours.

Challenge for the immediate future: extract physics!

→ light quark physics, contact with ChiPT

→ precision phenomenological computations

Entering a new era: dynamical light quarks

Various possible strategies:

- "Pure" → same fermion action for sea and valence quarks (tmQCD, chiral fermions).
- Mixed action approach (sea Wilson, chiral valence).
  - Techniques for chiral fermions ripe and ready for use, ability to reach even deep chiral regime.

    Giusti, Hernández, Laine, CP, Weisz, Wennekers, Wittig 2004-2006
- Detailed studies of systematics will be crucial.

Prepare for many exciting lattice QCD results!

Conclusions and outlook

- Lattice QCD reaching maturity: all sources of systematic uncertainties can be brought under control.

- State-of-the-art techniques with Wilson fermions provide benchmark quenched results.

    - tmQCD techniques extended to other cases ($K \to \pi\pi$, B_B).

CP, Sint & Vladikas 2004
Palombi, Papinutto, CP & Wittig 2006

- Light dynamical quarks running in machines worldwide: start of a new era.

Prepare for many exciting lattice QCD results!