Soluciones de Ejercicios de Métodos I

Curso 2004-2005. Hoja 5

1. .

2. .

3. .

   a) $x_1 = y,\ x_2 = t\,y'$

$$A = \begin{pmatrix} 0 & 1 \\ -\frac{c}{a} & 1-\frac{b}{a} \end{pmatrix}$$

4. .

   $\lambda_1 = 2 + 3i$ doble

$$\vec{v}_1 = \begin{pmatrix} -i \\ 3+3i \\ 0 \\ -1 \end{pmatrix},\ \vec{v}_2 = \begin{pmatrix} 3 \\ -10+9i \\ -i \\ 0 \end{pmatrix}$$

$$\vec{x}_1(t) = \vec{v}_1\, e^{\lambda_1 t} \equiv e^{2t}\,(\vec{y}_1 + i\vec{y}_2)$$

$$\vec{x}_2(t) = (\vec{v}_1\, t + \vec{v}_2)\, e^{\lambda_1 t} \equiv e^{2t}\,(\vec{y}_3 + i\vec{y}_4)$$

$$\vec{x}(t) = e^{2t}\, \textstyle\sum_{i=1}^{4}\, c_i\, \vec{y}_i(t)$$

5. .

   b) $\left|\frac{R_1}{L} - \frac{1}{CR_2}\right| > \frac{2}{\sqrt{LC}}$

   e) $I(t) = e^{-t}\,(\cos\sqrt{5}t - \frac{2}{\sqrt{5}}\sin\sqrt{5}t)$ A

   $V(t) = e^{-t}\,(2\cos\sqrt{5}t + \sqrt{5}\,\sin\sqrt{5}t)$ V

   f) $I(t_1) = 0 \;\rightarrow\; t_1 = 0.376$ s $\rightarrow\; V(t_1) = 3e^{-t_1} = 2.06\ V$

   $V(t_2) = 0 \;\rightarrow\; t_2 = 1.079$ s $\rightarrow\; I(t_2) = -\frac{3}{\sqrt{5}}e^{-t_2} = -0.456\ A$

6. .

$$x(t) = r_0\, \cos\omega t,\ \ y(t) = -r_0\, \sin\omega t$$

$$T = \tfrac{1}{2}m\vec{v}^2 + \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)$$

$$V = -\vec{\mu}\cdot\vec{B} = -\tfrac{q}{2m}\vec{L}\cdot\vec{B} = -\tfrac{1}{2}m\omega^2 r^2$$

$$E = T + V = \tfrac{1}{2}\vec{B}^2$$

$$x(t) = a\,(1-\cos\omega t),\ \ y(t) = a(\,\sin\omega t - \omega t)$$

$$\tfrac{1}{2}m\vec{v}^2 = m\omega^2 a^2\,(1-\cos\omega t)$$

$$V = -\vec{d}\cdot\vec{E} = -q\,a\,E\,(1-\cos\omega t)$$

$$E = T + V = \tfrac{1}{2}(\vec{E}^2 + \vec{B}^2)$$

7. .

a) $$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -(\gamma+\gamma_1) & \gamma \\ k & -k & \gamma & -(\gamma+\gamma_2) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -2 & 2 & -3 & 1 \\ 2 & -2 & 1 & -3 \end{pmatrix}$$

$$\vec{x}_1(t) = \vec{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \qquad \vec{x}_2(t) = \vec{v}_2\, e^{-2t} = \begin{pmatrix} 1 \\ 1 \\ -2 \\ -2 \end{pmatrix} e^{-2t}$$

$$\vec{x}_3(t) = \vec{v}_3\, e^{-2t} = \begin{pmatrix} 1 \\ -1 \\ -2 \\ 2 \end{pmatrix} e^{-2t}$$

$$\vec{x}_4(t) = (\vec{v}_3\, t + \vec{v}_4)\, e^{-2t} = \begin{pmatrix} t \\ -t \\ 1-2t \\ -1+2t \end{pmatrix} e^{-2t}$$

b) $x_1(t) = x_2(t) = \frac{v_0}{2}\,(1 - e^{-2t})$

$$x_1'(t) = x_2'(t) = v_0\, e^{-2t}$$

$$x(5\text{s}) = 4.9998\ \text{m},\ \ x(\infty) = 5\ \text{m}$$

8. .

c) $y_1(t) = \frac{v_0}{\sqrt{3}} \sin \sqrt{3}t, \quad y_2(t) = -\frac{v_0}{\sqrt{3}} \sin \sqrt{3}t$

d) $y_{1p}(t) = \frac{10}{3} \cos 2t, \quad y_{2p}(t) = \frac{10}{3} \cos 2t$

9. .

$x(t) = 3\cos t + \cos 3t$

$y(t) = 3\sin t - \sin 3t$

10. .

$$A = \begin{pmatrix} -20 & 10 & 0 & 0 & 0 & 0 & 0 \\ 10 & -20 & 10 & 0 & 0 & 0 & 0 \\ 0 & 10 & -20 & 10 & 0 & 0 & 0 \\ 0 & 0 & 10 & -20 & 10 & 0 & 0 \\ 0 & 0 & 0 & 10 & -20 & 10 & 0 \\ 0 & 0 & 0 & 0 & 10 & -20 & 10 \\ 0 & 0 & 0 & 0 & 0 & 10 & -20 \end{pmatrix}$$

$\lambda_1 = -38.2709, \quad \omega_1 = 6.1863$ rad/s, $\quad T_1 = 1.0157$ s

$\lambda_2 = -33.3826, \quad \omega_2 = 5.7778$ rad/s, $\quad T_2 = 1.0875$ s

$\lambda_3 = -26.1803, \quad \omega_3 = 5.1167$ rad/s, $\quad T_3 = 1.2280$ s

$\lambda_4 = -17.9094, \quad \omega_4 = 4.2320$ rad/s, $\quad T_4 = 1.4847$ s

$\lambda_5 = -10.0000, \quad \omega_5 = 3.1623$ rad/s, $\quad T_5 = 1.9869$ s

$\lambda_6 = -3.8197, \quad \omega_6 = 1.9544$ rad/s, $\quad T_6 = 3.2149$ s

$\lambda_7 = -0.4370, \quad \omega_7 = 0.6611$ rad/s, $\quad T_7 = 9.5042$ s

11. .

b) $\omega_1 = 15$ rad/s, $\omega_2 = 7.6$ rad/s

c) $v_1 = 237$ m/s, $v_2 = 121$ m/s

12. .

e) $I(t) = e^{-t}(2\cos t + 3\sin t)$ A

$V(t) = e^{-t}(\cos t - 5\sin t)$ V

f) $I(t) = 2e^{-t}(\sin t - \cos t + 1)$ A

$V(t) = 4e^{-t}(\cos t - 1)$ V