Soluciones de Ejercicios de Métodos I

Curso 2004-2005. Hoja 2

1. .

   i) $y(x) = \frac{x^2}{4} - \frac{x}{3} + \frac{1}{2} + \frac{1}{12x^2}, \qquad x > 0$

   ii) $y(x) = \frac{e^x+1-e}{x}, \qquad x > 0$

   iii) $y(x) = \frac{2x+1-\pi}{\sin x}, \qquad 0 < x < \pi$

   iv) $y(x) = \frac{\sin x - x\cos x}{x^2}, \qquad -\infty < x < \infty$

   v) $y(x) = -\frac{\cos 2x}{\cos x}, \qquad \pi/2 < x < 3\pi/2$

   vi) $y(x) = \frac{x^3+x+1}{x(x+2)}, \qquad -2 < x < 0$

   vii) $y(x) = e^{-x}\int_0^x \frac{e^s ds}{1+s^2}, \qquad -\infty < x < \infty$

   viii) $y(x) = -\frac{1}{3}(1-x^2) + \frac{7}{3}(1-x^2)^{-1/2}, \qquad -1 < x < 1$

2. b) No contradice el teorema de unicidad porque $\frac{\partial f}{\partial y}(x,y)$ no es contínua en $(2,-1)$.

3. $|y-2x|^{1/4}|y+2x|^{3/4} = C$

4. .

   i) $y(x) = \pm\left(\frac{2x^3}{3} + c\right)^{1/2}, \qquad y \neq 0, \;\; x > \left(-\frac{3c}{2}\right)^{1/3}$

   ii) $y(x) = \pm\left(\ln|1+x^2| + c\right)^{1/2}, \qquad y \neq 0, \;\; x > (e^{-c}-1)^{1/2}$

   iii) $y(x) = \frac{1}{c-\cos x}, \qquad x \neq \arccos c\ ; \qquad y(x) = 0$

   iv) $y(x) = \tan(x + x^2/2 + c), \qquad x + x^2/2 \neq (2n+1)\pi/2 - c$

   v) $y(x) = \frac{1}{2}\arctan(x + \sin x\cos x + c), \;\; \forall x; \qquad y(x) = (2n+1)\pi/4$

   vi) $y(x) = \sin(\ln x + c), \qquad x \neq 0, |y| < 1; \qquad y(x) = \pm 1$

   vii) $y^2 + 2e^y = x^2 + 2e^{-x} + c, \qquad y + e^y \neq 0$

   viii) $y^3 + 3y = x^3 + c, \qquad \forall x, \;\; \forall y$

5. .

i) $y^2 + 4x^2 = y_0^2, \qquad |x| \leq |y_0|/2, \ |y| \leq |y_0|$

ii) $y(x) = y_0/(1 - y_0 x^2), \qquad \begin{cases} y_0 > 0, & |x| < y_0^{-1/2} \\ y_0 < 0, & \forall x \end{cases}$

iii) $y(x) = y_0/\sqrt{1 + 2y_0^2 x}, \qquad x > -1/2y_0^2$

iv) $y(x) = \sqrt{\ln(1 + x^3)^{2/3} + y_0^2}, \qquad x > (e^{-3y_0^2/2} - 1)^{1/3}$

6. .

i) $N = 0$, inestable

ii) $N = 0$, inestable; $N = -a/b$, estable

iii) $N = 0$, inestable; $N = 1$, estable; $N = 2$, inestable

iv) $N = 0$, inestable

v) $N = 0$, estable

vi) $N = 0$, estable

vii) $N = 1$, semiestable

viii) $N = -1$, estable; $N = 0$, semiestable; $N = 1$, inestable

ix) $N = -1$, estable; $N = 0$, inestable; $N = 1$, estable

x) $N = 0$, estable; $N = b^2/a^2$, inestable

xi) $N = -2$, inestable; $N = 0$, semiestable; $N = 2$, estable

xii) $N = 0$, semiestable; $N = 1$, semiestable