Superstrings and Heterotic string phenomenology

1 Superstrings

1.1 Spacetime fermions vs worldsheet fermions

See discussion in sect 10 in [1].

In trying to connect string theory with the kind of physics observed in Nature, we have seen that compactification is able to solve the dimension problem of the bosonic string theory: how to get four-dimensional physics (at least at low energies) out of a theory which must propagate on a 26d spacetime.

A more difficult problem is that bosonic string theory does not contain spacetime fermions in its spectrum, and fermion fields are essential in our understanding of the real world. This (and also the closed string tachyon, etc) is enough to discard bosonic string theory as realized in Nature [1]

Happily there exist other string theories which are not the bosonic string theory. We are now advanced enough to understand that a string theory is basically defined by a 2d conformal field theory which provides the worldsheet action. What we are about to do is to construct a new kind of worldsheet theories, with Poincare invariance in d-dimensional spacetime, and which contain more fields than just the worldsheet scalars $X^\mu(\sigma, t)$. The resulting string theories have a spectrum of spacetime particles different from that in the bosonic string theory, and in particular they will turn out to contain spacetime fermions.

[1]Leaving aside the speculative possibility that bosonic string theory may contain fermions in its non-perturbative spectrum.

The basic idea is to supersymmetrize the 2d worldsheet theory. That is, we consider a 2d field theory with d worldsheet scalar fields $X^\mu(\sigma,t)$, and d worldsheet fermion fields $\psi^\mu(\sigma,t)$, which are their superpartners. In Polyakovs formulation one also has the worldsheet metric $g_{\alpha\beta}(\sigma,t)$ and now we also introduce its superpartner (which is a worldsheet gravitino). After gauge fixing these will disappear so we will not be very explicit about them.

String theories with worldsheet supersymmetry are called superstrings. They are just string theories with a different structure for the worldsheet action.

It is very important to notice that the 2d fields $\psi^\mu(\sigma,t)$ are fermions on the worldsheet (and so have anticommutation relations, etc in the 2d quantum field theory) but they transform as a vector under the d-dimensional spacetime Lorentz group, and so they behave as spacetime bosons. This makes sense because the Lorentz group is a global symmetry from the worldsheet viewpoint, and it commutes with the worldsheet supersymmetry, so 2d fields in the same supermultiplet must transform in the same way under the global symmetry.

So, the reason why superstrings contain spacetime fermions is not automatically because they contain fermions on the worldsheet. Indeed the connection is much more subtle and we will not study it until the detailed lectures.

Something similar happens with spacetime supersymmetry. The fact that superstrings have worldsheet supersymmetry does not imply that the spacetime spectrum of particles is supersymmetric. Several superstring theories DO have a spectrum of particles which is spacetime supersymmetric, but the way this arises is very subtle and follows from the so-called GSO projection. These are the most studied superstring theories, because they are well-behaved, for instance do not contain tachyons in their spectrum, so are stable. There also exist some superstrings which have a supersymmetric worldsheet theory, but are not supersymmetric in spacetime; very often they contain tachyons in their spectrum, so are not so much in control.

A common feature of all superstrings (and one which distinguishes them from the bosonic theory) is that, since we have modified the content of fields of the 2d worldsheet theory, the conformal anomaly changes, and the constraint on the number of dimension changes. The number of dimensions on which superstrings consistently propagate is 10. As usual, one uses compactification to construct theories with four-dimensional physics at low energies.

1.2 The different ten-dimensional superstring theories

Skipping many important details to be studied in coming lectures, here we would like to briefly describe the structure of the five superstring theories, which are also supersymmetric in spacetime (have a supersymmetric spectrum of spacetime particles).

For references on the structure of susy and sugra multiplets, see [2].

- **Type IIA superstring**

This is a theory of closed oriented strings.

Type IIA string theory has $N = 2$ (local) supersymmetry in ten dimensions, i.e. it is invariant under two Majorana-Weyl supercharges (of different chirality).

Its massless sector contains the following 10d bosonic fields: The graviton G_{MN}, a 2-form B_{MN}, the dilaton scalar ϕ; A 1-form A_M and a 3-form C_{MNP}. Their supersymmetric partners are basically some $N = 2$ $D = 10$ gravitinos of opposite chirality (and spin 3/2) and two spin-1/2 fermions of opposite chiralities.

We would like to remark that the p-form fields C_p are gauge potentials, namely all their interactions and couplings are invariant under the gauge transformations with gauge parameter given by a $(p-1)$-form Λ_{p-1}

$$C_p \to C_p + d\Lambda_{p-1} \tag{1}$$

The gauge invariant field strengths are given by

$$H_{p+1} = dC_p \tag{2}$$

The above matter content is the gravity supermultiplet of non-chiral $N = 2$ $D = 10$ supergravity. Indeed the low energy effective action of type IIA string theory is that of non-chiral $N = 2$ $D = 10$ supergravity, and its form is uniquely fixed by supersymmetry. It contains the Einstein term, the kinetic term for the p-forms and the dilaton, and their supersymmetric completion involving the fermions.

It is also useful to know that the degrees of freedom in a p-form gauge potential C_p can be encoded in a dual $(8-p)$-form $\hat{C}_{8-p}$ by Hodge-duality of their field strengths

$$H_{p+1} = *_{10d}\hat{H}_{9-p} \tag{3}$$

So the 1-form has a 7-form dual, and the 3-form has a 5-form dual.

- **Type IIB superstring**

This is a theory of closed oriented strings.

Type IIB string theory has $N = 2$ (local) supersymmetry in ten dimensions, i.e. it is invariant under two Majorana-Weyl supercharges (of SAME chirality).

Its massless sector contains the following 10d bosonic fields: The graviton G_{MN}, a 2-form B_{MN}, the dilaton scalar ϕ; A 2-form $\tilde{B}_{MN}$ and a 4-form A_{MNPQ} of self-dual field strength. Their supersymmetric partners are basically some $N = 2$ $D = 10$ gravitinos of SAME chirality (and spin 3/2) and two spin-1/2 fermions of SAME chiralities. The p-forms are gauge potentials.

The above matter content is the gravity supermultiplet of CHIRAL $N = 2$ $D = 10$ supergravity. Indeed the low energy effective action of type IIB string theory is that of CHIRAL $N = 2$ $D = 10$ supergravity, and its form is uniquely fixed by supersymmetry. It contains the Einstein term, the kinetic term for the p-forms and the dilaton, and their supersymmetric completion involving the fermions.

An important observation is that the theory is chiral, so in principle it may be ill-defined at the quantum level due to gravitational anomalies (i.e. diffeomorphism invariance of the classical theory may be violated at the quantum level, leading to violations of unitarity, etc and rendering the theory inconsistent). Happily a detailed computation of the anomaly shows that it vanishes (in a very nontrivial way) [3].

• **The two versions of Heterotic string theory**

This is a theory of closed oriented strings.

Heterotic string theory has $N = 1$ (local) supersymmetry in ten dimensions, i.e. it is invariant under one Majorana-Weyl supercharge.

Its massless sector contains the following 10d fields: The graviton G_{MN}, a 2-form B_{MN}, the dilaton scalar ϕ, plus fermion superpartners. They fill out a graviton supermultiplet of $N = 1$ $D = 10$ supergravity. In addition there are 496 gauge bosons A_M^a associated to generators of a gauge group, which is either $E_8 \times E_8$ or $SO(32)$ (so there are two different versions of heterotic string theory). These gauge bosons have fermionic partners (in the adjoint representation of the gauge group, gauginos), filling out vector multiplets of $D = 10$ $N = 1$ supersymmetry.

The low energy effective action is that of $N = 1$ $D = 10$ supergravity, coupled to $E_8 \times E_8$ or $SO(32)$ gauge vector multiplets. The supersymmetry and gauge symmetry uniquely fixed the form of the effective action. It contains the Einstein term, the kinetic term for the 2-form and the dilaton, and

Yang-Mills action for gauge bosons, and their supersymmetric completion involving the fermions.

An important observation is that the theory is chiral, so in principle it may be ill-defined at the quantum level due to gravitational and gauge anomalies. Happily a detailed computation of the anomaly shows that it vanishes (in a very nontrivial way), involving a novel mechanism (previously unknown in field theory), the so-called Green-Schwarz mechanism [4]. For the mechanism to work it is essential that the gauge group is one of the above mentioned.

- **Type I string theory**

This is a theory of closed and open unoriented strings. Unoriented means that the genus expansion includes non-orientable surfaces, like the Klein bottle or the Moebius strip, etc.

Type I string theory has $N = 1$ (local) supersymmetry in ten dimensions, i.e. it is invariant under one Majorana-Weyl supercharge.

Its massless sector contains the following 10d fields: The graviton G_{MN}, a 2-form B_{MN}, the dilaton scalar ϕ, plus fermion superpartners. They fill out a graviton supermultiplet of $N = 1$ $D = 10$ supergravity. In addition there are 496 gauge bosons A_M^a associated to generators of a gauge group, which $SO(32)$ (but NOT $E_8 \times E_8$). These gauge bosons have fermionic partners (in the adjoint representation of the gauge group, gauginos), filling out vector multiplets of $D = 10$ $N = 1$ supersymmetry.

The low energy effective action is that of $N = 1$ $D = 10$ supergravity, coupled to $SO(32)$ gauge vector multiplets. The supersymmetry and gauge symmetry uniquely fixed the form of the effective action. It contains the Einstein term, the kinetic term for the 2-form and the dilaton, and Yang-Mills action for gauge bosons, and their supersymmetric completion involving the fermions.

An important observation is that the theory is chiral, so in principle it may be ill-defined at the quantum level due to gravitational and gauge anomalies. Happily the anomaly cancels, also involving a version of the Green-Schwarz mechanism [5, 5].

This clearly shows that extra dimensions and supersymmetry and supergravity are ideas easily accommodated in the string theory setup. That (and the amazing self-consistency of the theory, namely the fact that it always leads to anomaly-free low-energy field theories) is the reason why lots of people got attracted into the study of these theories.

2 Heterotic string phenomenology

From the viewpoint of trying to reproduce the observed physics, many attempts were taken in the framework of Kaluza-Klein compactification in type II string theories. However, as discussed previously, it is difficult to reproduce chiral 4d fermions with the non-trivial gauge quantum numbers unless the original 10d theory contains elementary non-abelian gauge fields [6]. For that reason, compactification of other theories like type I or the heterotics is more promising.

In fact, most efforts centered in the study of heterotic theory. In a sense, if we study compactifications on curved spaces, where we use the low energy effective action, the type I theory looks very similar to the $SO(32)$ heterotic. Finally, there has been a traditional preference for the $E_8 \times E_8$ heterotic since it leads (in the simplest compactifications) to smaller gauge groups.

2.1 The picture of our world as a heterotic string compactification

Enough of a speculation! We would like to address what these constructions may have to do with the real world!

So, we conclude this brief review by describing the picture of our world as a heterotic string compactifications. This follows [7].

In order to obtain four-dimensional physics we need to take spacetime to be of the form $M_4 \times X_6$. The original 10d theory has a lot of supersymmetry: $D = 10$ $N = 2$ corresponds to 16 supercharges, the equivalent to $D = 4$ $N = 4$ supersymmetry. This amount of supersymmetry is too much to allow for 4d chiral fermions.

If X_6 is too simple, like a T^6, the supersymmetries are unbroken and we obtain a non-chiral theory. The reason why T^6 does not break supersymmetry is because it is flat, and has trivial holonomy group.

The holonomy group of a d-dimensional manifold (endowed with a metric) is defined by taking a vector, parallel-transporting it along a closed path, and finding the $SO(d)$ rotation relating the original vector and the final one. The set of all such roations for all possible closed paths is the holonomy group of the manifold (with the corresponding metric). For a torus, any vector comes back to itself (with no rotation at all) under parallel transport around any closed path. see figure 1.

For manifolds with non-trivial holonomy groups, there are topological obstructions to defining conserved supercharges globally [2], so the supersymmetry observed at low energies correspons only to the supercharges which can be defined globally.

A generic 6-dimensional manifold has holonomy $SO(6)$ and breaks all

[2]Similar to the impossibility of defining a global vector field in a 2-sphere, i.e. it is impossible to comb a 2-sphere without leaving hair whirlpools.

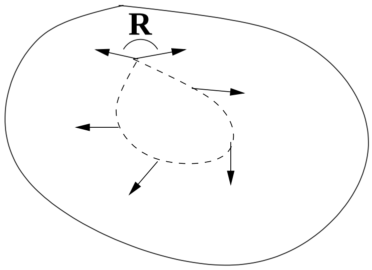

Figure 1: The holonomy group is given by the set of rotations R relating a vector and its image under parallel transport along a closed path, for all possible paths.

supersymmetries. Manifolds with holonomy in a proper subgroup of the generic holonomy group are known as special holonomy manifolds. They break some supersymmetries, but preserve some.

For heterotic string theory, if X_6 is chosen to have $SU(3)$ holonomy, (which is a subgroup of $SO(6)$), then the low energy theory in 4d has only $N = 1$ supersymmetry. As discussed in the first lecture, this is a phenomenologically desirable feature. Spaces of $SU(3)$ holonomy are called Calabi-Yau spaces, and compactification on them is often called Calabi-Yau compactification.

On the other hand, the original gauge group in heterotic string theory is very large, it has 496 generators. We should think about some way of breaking it. Happily there is a way of doing it in the process of compactification.

Consider that, in the same way as we consider a non-trivial background for the internal metric (curved internal space), we consider turning on a non-trivial background for the internal components of the gauge potentials. That is, we turn on a nontrivial profile for the fiels A_i^a, with i polarized in the internal directions in X_6, and a associated to generators in a subgroup H of the original group, say in $E_8 \times E_8$. In fancy language, we are considering a non-trivial gauge bundle (with structure group H) over the manifold X_6.

This choice is consistent with Poincare invariance in four dimensions. However, since it priviledges some direction in gauge space, the gauge group observed at low energies is not the full $E_8 \times E_8$. In fact, the 4d gauge group is given by those gauge transformations which leave the gauge background invariant. This is the group generated by generators commuting with the generators of H, and is called in group theory the commutant of H in $E_8 \times E_8$.

Moreover, it can be seen that the consistency of a Calabi-Yau compactification requires SOME internal gauge background to be turned on. This is interesting, because it forces the gauge group to be broken, althouth consistency does not force on us any specific choice of the subroup H.

A very popular choice is the so-called standard embedding, which amounts to choosing $H = SU(3)$. More specifically, it corresponds to setting the internal gauge connection to be equal to the Riemannian connection on X_6. With this choice, the commutant of $SU(3)$ in $E_8 \times E_8$ is $E_6 \times E_8$. This is a very exciting possibility, since E_6 has been considered as a possible group for grand unification. Taking slightly more involved choices for the gauge background it is possible to obtain even smaller groups, like $SU(5)$ of simply the Standard Model group.

The last ingredient that we would need is how to obtain chiral fermions charged under E_6 (or whatever other group we get in 4d). Amazingly the above ingredients (Calabi-Yau compactification and internal gauge bundle) are enough to provide chiral 4d fermions in the Kaluza-Klein reduction of the 10d gauginos. The resulting fermions transform naturally in the representation 27 of E_6 (or as $10 + \bar{5}$ of $SU(5)$, or standard fermion families of the standard model group).

The number of fermion families is given in terms of topological invariants of the internal manifold and the gauge bundle over it. For instance, for the standard embedding, it is given by the Euler number of X_6. The number of families is roughly speaking fixed by the number of (chiral) zero modes for a Dirac equation for the internal part of a 10d gaugino. So the different families are associated to different resonant modes of the 10d gaugino field in the internal X_6 space. B. Green describes this in a very poethic way [8]. It is possible (although not easy, it requires strong techniques in differential topology) to construct models where this number is 3.

The fact that the number of families is related to topological invariants is natural. In general one expects that, given a string compactification, the masses of light modes can vary if we make a small deformation of the configuration, like deforming the metric or the gauge background. However, the number of chiral families must be invariant under those deformations, because chirality protects fermions against getting Dirac masses. Hence, the number of chiral families is invariant under deformations of the metric or the gauge background, i.e. it is a topological invariant, which can be related to standard topological invariants of the manifold X_6 and the gauge bundle.

2.2 Phenomenological features and comparison with other proposals beyond the standard model

The lesson is that this picture, shown in figure 2, provides four-dimensional theories which are extremely close to the Standard Model.

Moreover, the description includes some very interesting ingredients of physics beyond the standard model

- Unification: All interaction arise from $E_8 \times E_8$, so at high enough energies $E \sim M_c$, when we start to be able to resolve the internal space, the original 10d gauge symmetry is restored and all interactions are unified. Of course, there is also unification with gravity, as in all

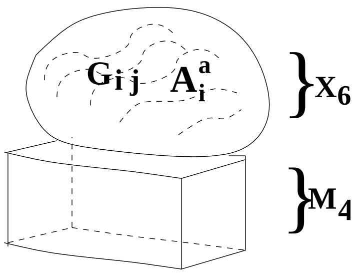

Figure 2: Schematic depiction of the compactification of heterotic string theory on a Calabi-Yau manifold (metric background) with non-trivial internal gauge bundle (gauge background).

string theories. Heterotic string theory also predicts gauge coupling unification at a scale $\sim M_s$

- Supersymmetry: Is a basic ingredient in this construction. The issue of supersymmetry breaking remains an open question
- Hidden sector: One attractive possibility is to break supersymmetry by strong coupling dynamics (gaugino condensation) in the untouched E_8. This sector is decoupled from the Standard Model one, with which it communicates only via gravitational interactions, it is a hidden sector. So it implements the idea of supersymmetry breaking in a hidden sector.
- Extra dimensions. Also essential in the construction. Notice that both gauge and gravitational interaction propagate in 10d, so this constructino cannot be used to realize the brane-world scenario (other constructions, not based in heterotic, will be studied later on).

There also remain different open questions, whose answer is not clear for the moment. These are the main problems in string phenomenology, to be solved perhaps by next-generation students like you!

- How to break supersymmetry? There exist proposals like gaugino condensation, etc.
- The moduli problem: Or how to get rid of the large number of massless scalars which exist in many compactifications in string theory (and whose vevs encode the parameters of the underlying geometry and gauge bundle (like sizes of the internal manifold, etc)).
- The vaccum degeneracy problem: Or the enormous amount of consistent vacua which can be constructed, out of which only one (if any at all) is realized in the real world. Is this model preferred by some energetic, cosmological, anthropic criterion? Or is it all just a matter of chance?
- The cosmological constant problem, which in general is too large once we break supersymmetry. Does string theory say anything new about this old problem?

References

[1] J. Polchinski, 'String theory', vol2, Cambridge univ press.

[2] J. Strathdee, 'Extended Poincare Supersymmetry', Int. J. Mod. Phys. A2 (1987) 273;
W. Nahm, 'Supersymmetries and their representations', Nucl. Phys. B135 (1978) 149.

[3] L. Alvarez-Gaume, E.Witten, 'Gravitational anomalies' Nucl.Phys.B234:269,1984.

[4] Michael B. Green, John H. Schwarz, 'Anomaly cancellation in supersymmetric D=10 gauge theory and superstring theory', Phys. Lett. B149: 117-122, 1984;

[5] Michael B. Green, John H. Schwarz, 'The hexagon anomaly in type I superstring theory', Nucl. Phys. B255: 93-114, 1985.

[6] E. Witten, 'Fermion quantum numbers in Kaluza-Klein theory', Published in Shelter Island II 1983:227

[7] P. Candelas, G. T. Horowitz, A. Strominger, E. Witten, 'Vacuum configurations for superstrings', Nucl. Phys. B258: 46-74, 1985

[8] B. R. Greene, 'The elegant universe: Superstrings, hidden dimensions, and the quest of the ultimate theory', New York, USA: Norton (1999) 448