Non-BPS D-branes in string theory

1 Motivation

In this lecture we present a new viewpoint on D-branes, arising from the Study of configurations of D-branes and anti-D-branes in string theory. The construction will imply some interesting insights into the meaning of tachyonic modes in string theory. Also, this viewpoint will lead to the construction of new stable non-BPS D-branes in string theory, which will allow to carry out a check of duality beyond supersymmetry. Some useful references for this talk are [1, 2].

2 Brane-antibrane pairs and tachyon condensation

2.1 Anti-D-branes

In analogy with particles and antiparticles in quantum field theory, every object in string theory has the corresponding antiobject, with equal tension but opposite charges. In particular, for every Dp-brane there exists a corresponding anti-Dp-brane state, denoted $\overline{Dp}$-brane, such that when they are put together they can annihilate each other into the vacuum.

$\overline{Dp}$-branes and Dp-branes have the same tension but opposite charges under the RR $(p+1)$-form. Note that this implies that $\overline{Dp}$-branes are also BPS states, which preserve half of the supersymmetry of the vacuum, but they preserve the supersymmetries broken by the Dp-branes, and vice versa. Namely, the supersymmetry generators $\epsilon_L Q_L + \epsilon_R Q_R$ unbroken by the presence of these objects in type II theory, are of the form

$$
\begin{aligned}
Dp \quad & \longrightarrow \epsilon_L = \Gamma^0 \ldots \Gamma p \epsilon_R \\
\overline{Dp} \quad & \longrightarrow \epsilon_L = -\Gamma^0 \ldots \Gamma^p \epsilon_R
\end{aligned} \tag{1}
$$

$\overline{Dp}$-branes are described, just as Dp-branes, as $(p+1)$-dimensional subspaces on which open strings are allowed to end. It is thus natural to consider what features distinguish Dp-branes and $\overline{Dp}$-branes , from the viewpoint of the 2d worldsheet. Equivalently, considering a configuration including both kinds of objects, what distinguishes open strings with both ends on the same kind of object, and open strings starting on branes and ending on antibranes (or viceversa). This is addressed in the following section.

2.2 Dp-$\overline{Dp}$-brane pair

Consider a configuration with a single Dp- and a single $\overline{Dp}$-brane in type II theory, with coincident worldvolumes along the directions $01\ldots p$. A prominent feature of this configuration is that it is non-supersymmetric. Namely there is no supercharge which is preserved by both the Dp- and the $\overline{Dp}$-brane. Another way to obtain the result is to notice that the state as a whole is not BPS: denoting T_p, Q_p the tension and charge of a Dp-brane, the state as a whole has tension $2T_p$ and charge 0. The tension of a BPS state in the topological sector of zero charge should be zero, hence the brane-antibrane pair is a non-BPS excited state. Notice that there is a clear BPS state in the zero charge sector of the theory, namely the type II

vacuum. Therefore we expect the non-BPS state given by the brane-antibrane pair to be unstable against decay to the vacuum, since both states have the same charges, and the vacuum is energetically favoured.

Let us compute the spectrum of open strings in the presence of the brane-antibrane pair. Clearly open strings with both ends on the Dp-brane (Dp-Dp

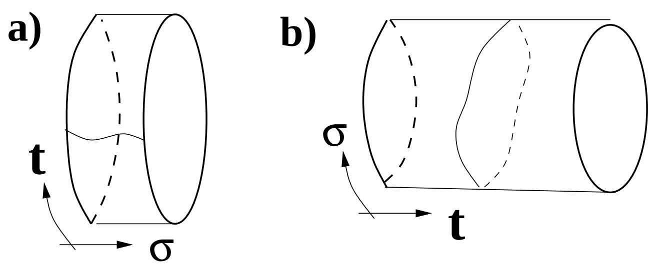

Figure 1: The annulus diagram regarded in the open and in the closed string channel.

strings) are not sensitive to the presence of the $\overline{Dp}$-brane, hence are quantized as usual. They lead to a $(p+1)$-dimensional $U(1)$ gauge boson and their superpartners with respect to the 16 supersymmetries unbroken by the Dp-brane. Similarly, $\overline{Dp}$-$\overline{Dp}$ open strings lead to a $(p+1)$-dimensional $U(1)$ gauge boson and its superpartners with respect to the 16 supersymmetries unbroken by the $\overline{Dp}$-branes (and which are the opposite of the above ones). Finally, we need to consider Dp-$\overline{Dp}$ and $\overline{Dp}$-Dp open strings. The boundary conditions are exactly the same as for the above sectors, namely Neumann for the directions $0,\ldots,p$ and Dirichlet for the directions $p+1,\ldots,9$. Hence, the Hilbert space of open string states, before any GSO projection, is the usual one. The lightest modes are

Sector	State	αM^2	Field
NS	$\|0\rangle$	-1	Scalars
	$\psi^{\mu}_{-1/2}\|0\rangle$	0	Gauge bosons + Scalars
R	8_C	0	Fermions
	8_S	0	Fermions

We now show that open-closed duality forces to choose the GSO projection in the Dp-$\overline{Dp}$ sector opposite to the usual one (namely, that in Dp-Dp or

$\overline{Dp}$-$\overline{Dp}$ sector). To see that, consider the annulus diagram, with two boundaries on Dp-branes, see figure 1a. Computing this amplitude in the open string channel, as a loop of Dp-Dp strings, we get

$$Z(T)_{pp} \simeq \frac{1}{2}\left(\mathrm{tr}_{NS} q^{N_F+E_0^F} + \mathrm{tr}_{NS}(q^{N_F+E_0^F}(-)^F\right) - \frac{1}{2}\left(\mathrm{tr}_R q^{N_F+E_0^F} + \mathrm{tr}_R(q^{N_F+E_0^F}(-)^F\right) =$$
$$= \frac{1}{2}\eta^{-4}\left(\vartheta\begin{bmatrix}0\\0\end{bmatrix}^4 - \vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^4\right) - \frac{1}{2}\eta^{-4}\left(\vartheta\begin{bmatrix}1/2\\0\end{bmatrix}^4 - \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}^4\right) \quad (2$$

This quantity can be rewritten as an amplitude of a closed string propagating for a time $T' = 1/(2T)$, by performing a modular transformation, leading to

$$Z(2T')_{pp} = \eta^{-4}\left(\vartheta\begin{bmatrix}0\\0\end{bmatrix}^4 - \vartheta\begin{bmatrix}1/2\\0\end{bmatrix}^4\right) - \eta^{-4}\left(\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^4 - \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}^4\right) \quad (3)$$

The amplitude in the closed channel describes the interaction between Dp-branes, via exchange of NSNS and RR fields, see figure 1b).

The amplitude for a Dp-$\overline{Dp}$-brane annulus, in the closed channel, differs from (3) in the sign of the terms corresponding to the exchange of RR fields. This is because of the opposite sign of the RR charge of the $\overline{Dp}$-brane with respect to the Dp-brane charge. Hence we obtain

$$Z(2T')_{p\bar{p}} = \eta^{-4}\left(\vartheta\begin{bmatrix}0\\0\end{bmatrix}^4 + \vartheta\begin{bmatrix}1/2\\0\end{bmatrix}^4\right) - \eta^{-4}\left(\vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^4 + \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}^4\right)$$

Going back to the open string channel, the annulus amplitude for Dp-$\overline{Dp}$ open strings going in a loop corresponds to

$$Z(T)_{pp} \simeq \frac{1}{2}\left(\mathrm{tr}_{NS} q^{N_F+E_0^F} - \mathrm{tr}_{NS}(q^{N_F+E_0^F}(-)^F\right) - \frac{1}{2}\left(\mathrm{tr}_R q^{N_F+E_0^F} - \mathrm{tr}_R(q^{N_F+E_0^F}(-)^F\right)$$

Hence we see that the signs imposing the GSO projection are flipped. Therefore, for Dp-$\overline{Dp}$ and $\overline{Dp}$-Dp open strings, the lightest modes are

Sector	State	αM^2	Field
NS	$\|0\rangle$	-1	Scalars
R	8_S	0	Fermions

These fields carry charges $\pm(+1,-1)$ under the $U(1)\times U(1)$ on the D- and anti-D-branes. The spectrum in these sectors is very different from the Dp-Dp and $\overline{Dp}$-$\overline{Dp}$ sectors. In particular, there is no enhanced gauge symmetry when branes and antibranes coincide. Note also that these sectors lead to a complex tachyon of the world-volume. We will discuss it in detail later on. Before that, let us simply mention that for branes and antibranes separated by a distance L in transverse space, the lightest mode in the NS sector has mass

$$M^2 = -\frac{1}{\alpha} + \frac{L^2}{(2\pi\alpha)^2} \tag{4}$$

The tachyonic mode develops for distances smaller than a critical distance of the order of the string scale, $L_c \leq 4\pi^2\alpha^{1/2}$. However, branes and antibranes initially at a distance larger than L_c feel a mutual attraction (since they have equal tension and opposite charge, hence attract both gravitationally and by RR Coulomb interactions), and tend dynamically to approach and decrease this distance until they reach the tachyonic regime.

2.3 Tachyon condensation

The meaning of this tachyon is that the configuration is unstable against annihilation of the brane-antibrane pair to the vacuum of type II theory. That is, the brane-antibrane pair corresponds to a configuration of the system which is sitting at the top of a potential. The negative mass square of the tachyon field simply means that the second derivative of the potential as a function of this field is negative at the top of the potential, see figure 2. One

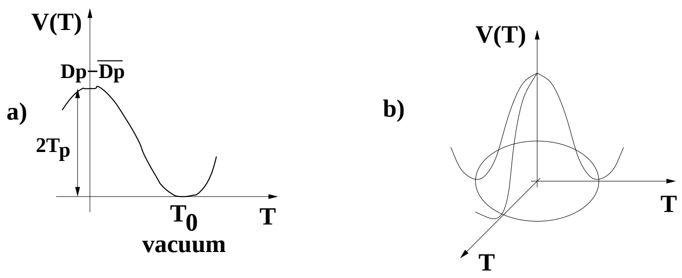

Figure 2: Two pictures of the tachyon potential for the brane-antibrane system.

should therefore let the tachyon roll down to the minimum of the potential, if it exists, to obtain a stable configuration. This process, by which the tachyon field acquires a vacuum expectation value T_0, minimizing the potential, is known as tachyon condensation.

A remarkable feature of this process is that there is a clear spacetime picture of its endpoint. The tachyon simply represents the instability of the brane-antibrane system against annihilation to the vacuum. This spacetime picture of the process of tachyon condensation implies that we know exactly the final state of this process: it is the vacuum of type II theory. So, although the initial state is non-supersymmetric, we can make exact statement about its fate after tachyon condensation..

Note that from the viewpoint of the world-volume theory, this process is similar in some respects to a Higgs mechanism. This is not completely precise, though. It is true that in the process a charged field (the tachyon) gets a vev, and breaks a gauge symmetry (the antisymmetric linear combination of the $U(1)$s on the D- and the anti-D-branes). However, the final state is the vacuum, where no open string states, and hence also the diagonal $U(1)$ linear combination, under which the tachyon is uncharged should also disappear. More strikingly, in the final state all open string modes of the initial state must be absent. Hence in the process of tachyon condensation an infinite number of fields disappear from the theory. This kind of processes have been successfully described only within the approach of string field theory.

Let us emphasize how remarkable it is the fact that we exactly know the final state of tachyon condensation. It leads to a number of exact statements about the properties of a non-supersymmetric brane-antibrane pair when the world-volume tachyon has a constant vev T_0. All of them are encoded in the statement that a brane-antibrane pair with a tachyon vev T_0 is indistinguishable from the vacuum. This is very surprising, for instance, the final state has an enhanced supersymmetry, it has zero energy, etc. The set of predictions (as well as several others to be studied later) following from this spacetime picture of tachyon condensation are known as Sens conjectures.

Also, very remarkably, we have succeeded in understanding the meaning of open string tachyons. In fact, we can extend this understanding to othe open string tachyons in string theory. For instance, tachyons in the open string sector of open bosonic string theory are now understood as an instability of bosonic D-branes to decay into the vacuum. This is consistent, since bosonic D-branes do not carry any conserved charge. Hence, we are recovering the result, briefly mentioned in the lecture on open strings, that open bosonic string theory is unstable into decay to purely closed bosonic string theory, with no open string sector at all.

3 D-branes from brane-antibrane pairs

In this section we discuss other processes of tachyon condensation in brane-antibrane systems, where the final state is not the vacuum, but a lower-dimensional D-brane.

3.1 Branes within branes

For this section, see [3]. Recall that a Dp-brane in charged not only under the RR $(p+1)$-form, but also Under other lower-degree RR forms, if the world-volume gauge bundle is non-trivial. For instance, consider the Wess-Zumino couplings for a D3-brane

$$S_{WZ} = \int_{D3} C_4 + \int_{D3} C_2 \wedge \mathrm{tr}\, F + \int_{D3} C_0 \wedge \mathrm{tr}\, F^2 \tag{5}$$

Consider a world-volume gauge bundle with non-zero first Chern class, i.e. $\mathrm{tr}\, F$ is non-trivial on the D3-brane world-volume. This intuitively corresponds to turning on a magnetic field along two of the directions, say 23, in the D3-brane volume, with total integral e.g. $\int_{23} F = 1$. The above couplings imply that the D3-brane is charged under the RR 2-form C_2, or that we are dealing with a bound state of a D3-brane and a D1-brane (with volume along 01). In a sense, the system can be thought of as a D3-brane with a D1-brane diluted in its volume [1].

Similarly, a non-trivial $\mathrm{tr}\, F$ on a general Dp-brane induces D$(p-2)$-brane charge, a non-trivial second Chern class (or instanton number) $\mathrm{tr}\, F^2$ induces non-trivial D$(p-4)$-brane charge, etc.

3.2 D-branes from brane-antibrane pairs

Consider a $\overline{D3}$-brane with trivial world-volume gauge bundle, and a D3-brane with one unit of induced D1-brane charge, see figure 3. The complete system has zero D3-brane charge, one unit of D1-brane charge, and non-zero 3-brane tension (slightly larger than but around $2T_3$).

[1] Indeed this is quite precise. Starting with a configuration of coincident D3- and D1-branes there is a dynamical process diluting the D1-brane as world-volume gauge field strength on the D3-brane.

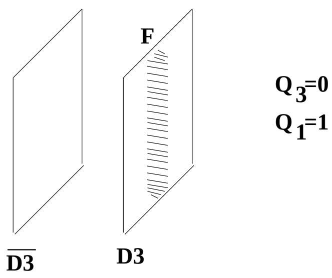

Figure 3: Brane-antibrane system with induced lower-dimensional brane charges.

Clearly the state is non-supersymmetric. One way to understand this is to note that there exists a state with the same charges and much less energy, namely a BPS D1-brane. Hence we expect, from the spacetime viewpoint, that the initial system is unstable to decay into a D1-brane state. Notice that decay to the vacuum is not consistent with charge conservation. Heuristically, the decay to the D1-brane state can be understood by considering the magnetic field to be localized in a more or less compact core in the directions 23, and translationally invariant along 01. Asymptotically away from the core, we just have a D3-$\overline{D3}$-brane pair, with no magnetic field density, so the system will suffer tachyon condensation annihilating them in the asymptotic region. Near the core, the magnetic field changes things, and annihilation leads an object compactly supported in 23, namely the D1-brane.

From the viewpoint of the 4d world-volume, the above system is described as follows. In a D3-$\overline{D3}$-system, we have a gauge group $U(1)^2$, and a complex scalar T with charge $(+1,-1)$, with a Mexican hat potential shown in figure 2b. Note that gauge invariance implies that the potential is function of the modulus of T, $V(|T|)$. The diagonal $U(1)$ subgroup decouples and will be irrelevant for the following discussion. This field theory has soliton solutions, which correspond to topologically non-trivial world-volume field configurations. Finite energy solitons must have a tachyon field asymptoting to the value $|T| = T_0$. Considering configurations which are translationally invariant in 01, $T = T(x^2, x^3)$, the tachyon field taken at the $\mathbf{S^1}$ at infinity in 23 defines a map from the spacetime $\mathbf{S^1}$ to the set of minima of the potential, which is also an $\mathbf{S^1}$. Topologically inequivalent solitons correspond to topologically inequivalent tachyon field configurations, which correspond to topologically inequivalent maps $\mathbf{S^1} \to \mathbf{S^1}$. The latter are classified by the homotopy group $\Pi_1(\mathbf{S^1}) = \mathbf{Z}$, i.e. there are an infinite number of inequivalent solitons, characterized by an integer, known as winding number of the above map. A simple example is provided by the winding one configuration. Defining $z = x^2 + ix^3$, the tachyon profile for the corresponding soliton is

$$T(z) = T_0 \frac{z}{|z|} \tag{6}$$

Representing the complex value of the tachyon by an arrow, the field configuration is of the hedgehodge form shown in figure 4. In order to have a finite energy configuration, we also need to turn on a non-trivial gauge field, so that the covariant derivatives approach zero fast enough as $|z| \to \infty$. This gauge field is such that there is a non-trivial first Chern class over 23, $\int_{23} F = 1$. The whole field configuration is known as vortex, and is the world-volume description of the tachyon condensed configuration. Indeed, asymptotically the system approaches the configuration of a D3-$\overline{D3}$-brane with a tachyon vev of T_0, hence describing asymptotic annihilation. Near the core, the tachyon value is approximately zero, and no annihilation is implied. In fact, near the core we have a D3-$\overline{D3}$ system with non-condensed tachyon; hence, open strings are allowed to end in the near core region of the above system. The system described an object localized in 23, charged under C_2 and on which open strings can end. This is clearly a D1-brane, which we have construced as a bound state of a higher-dimensional brane-antibrane pair.

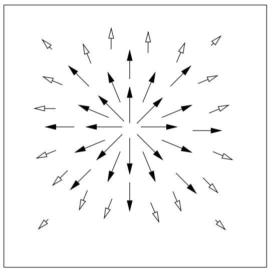

Figure 4: Picture of the hedgehodge configuration for the tachyon field in the vortex solution.

The above construction suggests the construction of D-branes as bound states, upon tachyon condenstation, of higher dimensional brane-antibrane pairs. This is a surprising new viewpoint, where D-branes are regarded as solitons on the world-volume of brane-antibrane pairs.

We can use a similar strategy to construct other D-brane states, in particular unstable Non-BPS Dp-branes in type II theory (with p even for IIB and odd for IIA). For instance, consider a D3-$\overline{D3}$ pair, with a tachyon profile corresponding to a kink in one dimension, say 3, see figure 5. The field configuration is localized in a compact region in x^3, and has trivial field strengths.

This world-volume configuration is not topologically stable, the kind can be continuously unwound into a trivial configuration. This implies that the resulting D2-brane, denoted $\widehat{D2}$-brane, is unstable (against decay to the vacuum), which is consistent since it carries no conserved charges. We would like to mention two further facts on these non-BPS branes: First, they admit a microscopic description, as subspaces on which open strings end. In this situation, the fact that these D-branes do not carry RR charges implies, by open-closed duality, that open strings stretching between non-BPS D-branes

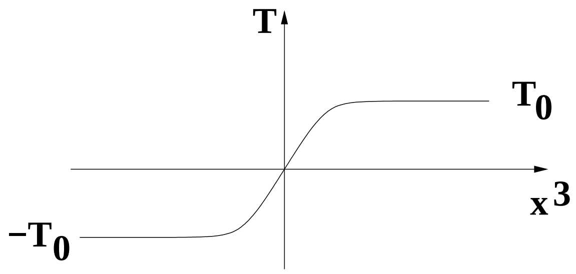

Figure 5: Picture of the tachyon profile in the kink configuration..

of this kind have a world-volume spectrum with no GSO projection. This spectrum is easily obtained, and in particular contains a real tachyon. Second, a further kink configuration on this world-volume tachyon corresponds to the condensation of an unstable non-BPS Dp-brane to a BPS D$(p-1)$-brane, of the usual kind (these relations are known as descent relations).

4 D-branes and K-theory

Let us generalize the idea that D-branes are constructed as bound states of higher-dimensional brane-antibrane pairs, upon tachyon condensation. The latter statement means that, at the topological level, states which differ by processes of creation/annihilation of brane-antibrane pairs must be considered equivalent.

Let us apply these ideas to type IIB theory on a spacetime X, and try to classify all D-brane charges. Namely, consider a type IIB configuration with n D9-$\overline{D}$9-brane pairs. Note that this is consistent, since the tadpole for the RR 10-form C_{10} generated by the D9- and the $\overline{D}$9-branes cancel each other [2]. In general, the D9-branes carry a world-volume $U(n)$ gauge bundle E, and

[2]Recall that the only inconsistency in coupling 10d Poincare invariant open string

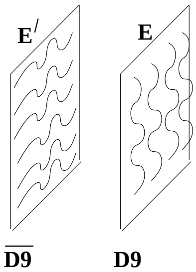

Figure 6: Brane-antibrane pairs with general world-volume gauge bundles.

the $\overline{D9}$-branes carry another $U(n)$ gauge bundle E. Hence, different D-brane states or D-brane charges are classified by pairs of bundles (E, E). However, configurations that differ by the nucleation of D9- and $\overline{D9}$-branes, both with world-volume gauge bundle H, must be considered topologically equivalent. Therefore the set of topologically inequivalent D-brane states is given by the set of pairs of bundles (E, E), modulo the equivalence relation

$$(E, E) \quad \simeq \quad (E \oplus H, E \oplus H) \tag{7}$$

The set of pairs of topological bundles up to this equivalence relation is a finitely generated group, known as (complex) K-theory group of the space-time X, denoted $K(X)$.

Let us describe the classification of type IIB charged D-branes in flat 10d space from this viewpoint. If we are interested in p-brane states (i.e. states with Poincare invariance in $(p+1)$ dimensions)the bundles are non-trivial only over the $(9-p)$-dimensional transverse space. Also, we are interested

sector to oriented type IIB theory arose from RR tadpole cancellation. Note also that the configuration with equal number of D9 and $\overline{D9}$-branes is regarded not as a new string theory, but as an excited state of type IIB theory (connected to the vacuum via tachyon condensation).

in bundles with compact support, so that the resulting states are localized in $\mathbf{R^{9-p}}$. See figure 6. Bundles with compact support on $\mathbf{R9-p}$ can be described as general bundles over $\mathbf{S^{9-p}}$. The corresponding K-theory groups have been computed by mathematicians and read

$$\begin{aligned} K(\mathbf{S^{9-p}}) &= \mathbf{Z} \quad \mathbf{p}\,\text{odd} \\ &= 1 \qquad p\,\text{even} \end{aligned} \tag{8}$$

Hence type IIB theory contains stable Dp-branes for p odd. These branes are stable since their charge, classified by the K-theory class, which is topological, forbids their decay to the vacuum. The fact that the K-theory classes are $\mathbf{Z}$ valued implies that their charge is additive. In fact, these are the familiar BPS we already know about, and their charge is the charge under the RR $(p+1)$-form field. Hence the classification of D-brane states using K-theory agrees with the classification using cohomology (namely computing the charge of a D-brane state as the flux of a certain form field over a cycle surrounding the D-brane) [3] . This is not so surprising, since there is a natural map from K-theory to cohomology, which to each K-theory class represented by a pair of bundles (E,E) it assigns the cohomology class

$$(E,E) \longrightarrow \mathrm{ch}(E) - \mathrm{ch}(E) \tag{9}$$

where $\mathrm{ch}(E)$ is the Chern character, defined by

$$\mathrm{ch}(E) = \mathrm{tr}\ e^{F/2\pi} = 1 + \frac{1}{2\pi}\mathrm{tr}\,F + \frac{1}{8\pi^2}\,\mathrm{tr}\,F^2 + \ldots \tag{10}$$

The Chern character is additive, $\mathrm{ch}(E\oplus) = \mathrm{ch}(E) - \mathrm{ch}(H)$, hence the above map is independenent of the representative of the K-theory class. Finally,

[3]Notice that we are classifying topological D-brane states. In particular, the unstable D-branes of type IIB theory do not appear in this classification since they can decay to the vacuum, namely are topologically equivalent to it.

notice that the Chern character enters in the Wess-Zumino couplings on the world-volume of a D-brane, hence it carries the information on the induced D-brane charges under the RR p-form fields.

There are situations, however, where the above mapping is not injective. Namely there may are situations where there exist non-trivial K-theory classes whose Chern character vanishes. Namely, there exist bundles which are topologically non-trivial but whose characteristic classes all vanish. This implies the existence of D-branes which are stable (since they carry a non-trivial topological quantum number) but are uncharged under the RR fields.

The simplest example of this kind is provided by type I theory. The classification of D-branes can be carried out as above. Namely, introduce n additional D9-$\overline{D9}$ over the vacuum of type I theory (note that this is consistent, since the total system contains $(n+32)$ D9-branes and n $\overline{D9}$-branes, leading to a total RR 10-form tadpole cancelling that of the O9-planes). The D9- and $\overline{D9}$-branes carry $SO(n)$ bundles E, E. D-brane configurations are topologically classified by pairs of bundles (E, E) modulo the equivalence relation (7). The resulting set is a group known as the real K-theory group of the spacetime X $KO(X)$.

Let us classify type I D-brane charges in flat space. As before, we need to compute the groups $KO(\mathbf{S^{9-p}}$, which have been computed by mathematicians. We obtain the following sets of D-branes

$$\begin{array}{rlcl} KO(\mathbf{S^1}) & = \mathbf{Z_2} & \to & \widehat{D8} \\ KO(\mathbf{S^2}) & = \mathbf{Z_2} & \to & \widehat{D7} \\ KO(\mathbf{S^4}) & = \mathbf{Z} & \to & D5 \\ KO(\mathbf{S^8}) & = \mathbf{Z} & \to & D1 \\ KO(\mathbf{S^9}) & = \mathbf{Z_2} & \to & \widehat{D0} \\ KO(\mathbf{S^{10}}) & = \mathbf{Z_2} & \to & D\widehat{(-1)} \end{array} \qquad (11)$$

Beyond the familiar BPS D1- and D5-branes, the K-theory classification implies the existence of non-BPS D8-, D7-, D0- and D(-1)-brane charges. They are completely uncharged under the RR fields, however they carry a non-trivial $\mathbf{Z_2}$ charge and cannot decay into the vacuum.

We would like to conclude with some comments

• For type IIA theory, there also exists a K-theory classification of D-brane charges. It is based on classifying bundles over spacetime filling unstable $\widehat{D9}$-branes. The relevant K-theory groups are known as (complex) reduced K-theory groups $K^{-1}(X)$. There is a relation between these and the type IIB groups, which is consistent with T-duality. For instance in 10d space, we have $K^{-1}(\mathbf{S^n}) = \mathbf{K}(\mathbf{S^{n-1}})$. This leads to the familiar set of BPS states of type IIA theory.

• The above construction is valid for D-branes, since they naturally carry world-volume gauge bundles. It is still an open issue to extend this kind of classification scheme to other theories without D-branes, like heterotic theories or M-theory, and to other objects, like NS5-branes.

5 Type I non-BPS D-branes

We have seen that type I contains different non-BPS D-branes with non-trivial topological charge. Since these charges are topological, states with these charges must exist in the spectrum for all values of the moduli (although their microscopic description may change in between). This allows to test string duality for this particular class of non-BPS states, i.e. test string duality beyond supersymmetry [4]

[4]Note that the lack of the BPS property however implies that we do not have much control over properties like the tension of the object, as the moduli change. Hence the tests are much less exhaustive than for supersymmetric states.

5.1 Description

The $\widehat{D0}$-brane

Altough we have described it starting with D9-$\overline{D9}$-brane pairs, the simplest construction starts from a D1-$\overline{D1}$-brane pair in type I theory. The world-volume gauge group is $\mathbf{Z_2} \times \mathbf{Z_2}$. The fact that this gauge group is discrete ensures that a kink configuration for the world-volume tachyon cannot be unwound, and hence describes tachyon condensation to a stable state. This is the stable $\widehat{D0}$-brane of type I theory [5]. The fact that it carries a $\mathbf{Z_2}$ charge means that two of these states can annihilate to the vacuum. This is understood in the D1-$\overline{D1}$-brane pair because two kinks can unwind to the trivial configuration for the world-volume tachyon, describing decay to the vacuum.

There is a microscopic description for the type I $\widehat{D0}$-brane, as a 1d subspace on which open strings can end. Such open strings have no GSO projection, in agreement (via open-closed duality) with the fact that they carry no RR charges. The light spectrum on the world-volume of a stack of n $\widehat{D0}$-branes is as follows. In the 00 sector, there is no GSO projection. The states are computed as usual (with some subtlety due to the fact that

there are not enough NN directions to use the light-cone gauge), and projected into Ω-invariant states. In the NS sector, there are massless $SO(n)$ gauge bosons, and 9 scalars in the representation $\square\!\square$; there are also a world-volume real tachyon, transforming in the representation $\begin{smallmatrix}\square\\\square\end{smallmatrix}$, so it is absent for $n = 1$ (in which case the system is stable). In the R sector, the groundstates give rise to fermions in the $\square\!\square + \begin{smallmatrix}\square\\\square\end{smallmatrix}$ representation. In the $09 + 90$ sector, the NS states are massive, while in the R sector the groundstate gives rise to

[5] Equivalently, one can describe the type I $\widehat{D0}$-brane as the type IIB $\widehat{D0}$-brane, modded out by Ω, which projects out the world-volume tachyon of the latter.

massless 1d fermions in the 32 of the D9-brane $SO(32)$ group.

For $n = 1$ we have a stable particle, with worldline described by the above fields. It has nine worldline bosons, so the particle propatates in 10d. On the other hand, there are worldline zero modes, which imply that in the quantum theory the particle belongs to a multiplet. Quantization of fermion zero modes in the 00 sector gives a 256-fold multiplicity, implying the particle state belongs to a non-BPS multiplet. Quantization of fermion zero modes in the 09 sector imply that the particle transforms in a non-trivial representation of $SO(32)$, in particular a 2^{15}-fold dimensional chiral spinor representation (there also appears the spinor representation of opposite chirality, but it is eliminated by the world-volume $\mathbf{Z_2}$ gauge group).

The fact that type I contains states in the spinor representation of a given chirality implies that its spacetime gauge group is globally not $SO(32)$. All perturbative and non-perturbative states are consistent with a gauge group $Spin(32)/Z_2$ (where $Spin$ allows the existence of spinor representations, and $\mathbf{Z_2}$ forbids the existence of spinors of one chirality and of states in vector representation).

The $\widehat{D8}$-brane

There exists a type I $\widehat{D8}$-brane described microscopically as a 9d subspace on which open strings end, and which carries the correct K-theory charge. However, the brane contains a world-volume tachyon arising in the sector of open strings stretching from the brane to the background D9-branes. This tachyon implies the non-BPS brane is unstable to decay, but not to the vacuum (which is forbidden by charge conservation) but to a different configuration carrying the same charge. The latter configuration is a non-trivial bundle on the D9-branes where there is a $\mathbf{Z_2}$ Wilson line on one of the D9-branes [6]

[6] This Wilson line is topological in the sense that it is an element in $Spin(32)/\mathbf{Z_2}$ but

The $\widehat{D(-1)}$-brane

The $\widehat{D(-1)}$-brane can be constructed starting from a type I D1-$\overline{D1}$-brane pair with a vortex configuration for the world-volume tachyon. Equivalently it can be described as a D(−1)-$\overline{D(-1)}$-brane pair of type IIB theory, modded out by Ω, which exchanges the D(−1) and the $\overline{D(-1)}$-brane, and eliminates the world-volume tachyon. The latter description provides a simple microscopic description for the type I D-instanton, but we will skip its detailed discussion.

The $\widehat{D7}$-brane

This can be described as a type IIB D7-$\overline{D7}$-brane pair, modded out by Ω, which exchanges the objects in the pair, and eliminates the world-volume tachyon in the $7\overline{7}$ sector. The tachyon in the 79 and $\overline{7}9$ sectors however survive, implying that the system is unstable against decay, not to the vacuum but to a non-trivial bundle on the background D9-branes. The bundle is described by two $\mathbf{Z_2}$ Wilson lines which commute up to a sign in $SO(32)$, namely which commute in $Spin(32)/\mathbf{Z_2}$ but not in $SO(32)$.

5.2 Heterotic/type I duality beyond supersymmetry

Non-BPS states in type I theory, which are nevertheless stable due to charge conservation, must exist not only at weak coupling (where we have provided a microscopic description), but at all values of the coupling. This implies that they lead to results which can be extrapolated to strong coupling, and be compared with properties of the heterotic theory.

The perturbative group of type I theory is $O(32)$. However we have seen that the global structure of the group is $Spin(32)/\mathbf{Z_2}$, since the theory contains states that transform in a chiral spinor representation, and states

not of $SO(32)$.

described by gauge configurations which do not exist in $SO(32)$. Finally, the non-BPS D-instanton plays a crucial role in describing the change in gauge group. Namely, it is not invariant under large gauge transformations in $SO(32)$. The true gauge group consistent with all non-BPS states of the theory is in fact the appropriate one to agree with the heterotic theory upon type I / heterotic duality. This is a first non-trivial result of non-BPS type I theory.

An even more remarkable check is that the particles described as type I $\widehat{D0}$-branes provide states that transform in a chiral spinor representation of the spacetime gauge group. By duality, heterotic theory should contain some states with the same basic properties, namely same gauge representation, and same non-BPS supermultiplet. Indeed, the $Spin(32)/\mathbf{Z_2}$ heterotic theory does contain states with these properties, they are given by massive perturbative heterotic states, with left-handed internal 16d momentum

$$P = \frac{1}{2}(\pm, \ldots, \pm) \quad , \quad \# \; - \; = \text{even} \tag{12}$$

This is Sens great idea on using these states to test string duality beyond supersymmetry.

6 Final comments

In this lecture we have studied a beautiful set of ideas, concerning a new viewpoint on D-branes. They have been widely generalized to more involved configurations, like orbifolds and orientifolds.

The construction of D-branes as bound states of higher-dimensional brane-antibrane pairs has allowed us to make precise exact statements on tachyon condensation processes in non-supersymmetric systems. Finally, these results have provided a new tool to test and partially confirm string duality beyond supersymmetry.

There are many other applications of these ideas to other related contexts. For instance the study of condensation of tachyons of other kinds (most interestingly the study of closed string tacyons is an open issue), or the application of antibranes and non-BPS D-branes as a source of supersymmetry breaking in model building. We leave these questions for the interested reader.

References

[1] E. Witten, 'D-branes and K theory', JHEP 9812 (1998) 019, hep-th/9810188.

[2] A. Sen, 'NonBPS states and Branes in string theory', hep-th/9904207.

[3] M. R. Douglas, 'Branes within branes', hep-th/9512077