Brane-worlds

1 Introduction

We have seen that branes in string theory may lead to gauge sectors localized on their world-volumes. This can be exploited, as we did in previous lecture, to take a decoupling limit where dynamics reduces to gauge field theory, and try to use string theory tools to gain new insights into gauge field theory dynamics.

In this lecture we would like to center on a different application of branes and their gauge sectors. There exist string theory or M-theory *vacua* with gauge sectors localized on the volume of branes, or on lower-dimensional subspaces of spacetime. For intance, in Horava-Witten compactifications, or in type I' theory (or its T-dual versions). These vacua can be regarded as a new possible setup in which to construct four-dimensional models with physics similar to that of the observed world, i.e. gravitational and gauge interactions, which charged chiral fermions. In this lecture we discuss different possible constructions containing gauge sectors that come close enough to the features of the Standard Model. Their main novelty is that gravitaional interactions and gauge interactions propagate over different spaces. See figure 1. This implies a different scaling of their interaction strength as functions of the underlying parameters/moduli of the model.

Heterotic model building

To understand better this point, recall the setup of compactifications of heterotic string theory on Calabi-Yau manifolds $\mathbf{X_6}$. The 4d gauge group is given by the commutant of H in G (namely the elements of G commuting with H), where G is the 10d $E_8 \times E_8$ or $SO(32)$. Thus, 4d gauge interactions are inherited from 10d ones, and so propagate all over 10d spacetime. Fig.

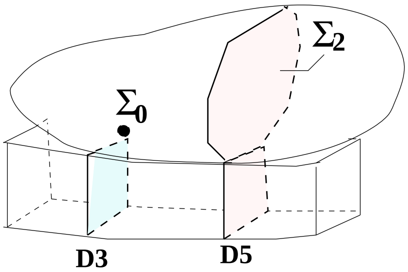

Figure 1: In compactifications with D-branes, the gauge sectors like the Standard Model could propagate just on a lower-dimensional subpace of spacetime, e.g. the volume of a suitable set of D-branes, like any of the shaded areas.

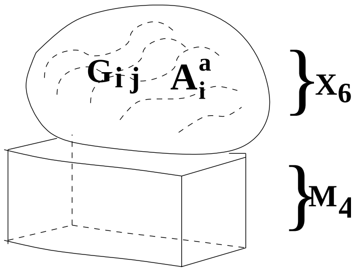

Figure 2: Picture of heterotic string compactification.

2 shows configurations of this kind.

A very important property in this setup is the value of the string scale, which follows form analyzing the strength of gravitational and gauge interactions, as we quickly review. The 10d gravitational and gauge interactions have the structure

$$\int d^{10}x \, \frac{M_s^8}{g_s^2} \, R_{10d} \quad ; \quad \int d^{10}x \, \frac{M_s^6}{g_s^2} \, F_{10d}^{\,2} \tag{1}$$

where M_s, g_s are the string scale and coupling constant, and R_{10d}, F_{10d} are the 10d Einstein and Yang-Mills terms. Powers of g_s follow from the Euler characteristic of the worldsheet which produces interactions for gravitons and gauge bosons (the sphere). Upon Kaluza-Klein compactification on $\mathbf{X_6}$, these interactions reduce to 4d and pick up a factor of the volume V_6 of $\mathbf{X_6}$

$$\int d^4x \, \frac{M_s^8 \, V_6}{g_s^2} \, R_{10d} \quad ; \quad \int d^4x \, \frac{M_s^6 \, V_6}{g_s^2} \, F_{10d}^{\,2} \tag{2}$$

From this we may express the experimental 4d Planck scale and gauge coupling in terms of the microscopic parameters of the string theory configuration

$$M_P^2 \, = \, \frac{M_s^8 \, V_6}{g_s^2} \, \simeq \, 10^{19} \, \mathrm{GeV} \quad ; \quad \frac{1}{g_{YM}^2} \, = \, \frac{M_s^6 \, V_6}{g_s^2} \, \simeq \, \mathcal{O}(.1) \tag{3}$$

From these we obtain the relation

$$M_s \, = \, g_{YM} \, M_P \, \simeq \, 10^{18} \, \mathrm{GeV} \tag{4}$$

which implies that the string scale is necessarily very large in this kind of constructions. The key points in the derivation are that all interactions propagate on the same volume, and their strengths have the same dilaton dependence.

Brane-world constructions

Models where gravitational and gauge interactions propagate on different spaces are knonw as brane-worlds, since fields in the Standard Model (those that make up the observable world) are localized on some brane (or in general, some subspace of spacetime. In these constructions 4d gauge and gravitational interaction strength have a different dependence on the internal volumes.

The prototypical case [1] is provided by a compactification of type II theory (or some orientifold quotient thereof) on a 6d space $\mathbf{X_6}$, with a gauge sector

[1] The following analysis does not apply directly to Horava-Witten compactifications, see [1] for the corresponding discussion.

localized on the volume of a stack of Dp-branes [2] wrapped on a $(p-3)$-cycle $\Pi_{(p-3)}$, with $\Pi_{(p-3)} \subset \mathbf{X_6}$. Namely, the $(p+1)$-dimensional world-volume of the Dp-brane is of the form $M_4 \times \Pi_{(p-3)}$. Before compactification, gravitational and gauge interactions are described by an effective action

$$\int d^{10}x \, \frac{M_s^8}{g_s^2} \, R_{10d} + \int d^{p+1}x \, \frac{M_s^{p-3}}{g_s} \, F_{(p+1)d}^2 \tag{5}$$

where the powers of g_s follow from the Euler characteristic of the world-sheet which produces interactions for gravitons (sphere) and for gauge bosons (disk).

Upon compactification, the 4d action picks up volume factors and reads

$$\int d^4x \, \frac{M_s^8 V_6}{g_s^2} \, R_{4d} + \int d^4x \, \frac{M_s^{p-3} V_{\mathbf{\Pi}}}{g_s} \, F_{4d}^2 \tag{6}$$

This allows to read off the 4d Planck mass and gauge coupling, which are experimentally measured.

$$\begin{aligned} M_P^2 &= \frac{M_s^8 V_{X_6}}{g_s^2} \simeq 10^{19}\,\mathrm{GeV} \\ 1/g_{YM}^2 &= \frac{M_s^{p-3} V_{\mathbf{\Pi}}}{g_s} \simeq 0.1 \end{aligned} \tag{7}$$

If the geometry is factorizable, we can split $V_{X_6} = V_{\mathbf{\Pi}} V_{\perp}$, with $V_{\perp}$ the transverse volume, and obtain

$$M_P^2 \, g_{YM}^2 = \frac{M_s^{11-p} V_{\perp}}{g_s} \tag{8}$$

This shows that it is possible to generate a large Planck mass in 4d with a low string scale, by simply increasing the volume transverse to the brane, or tuning the string coupling. In particular, it has been proposed to lower

[2] For the moment, the D-brane configuration is simplified for convenience. Later on we will see detailed configurations leading to interesting world-volume spectra.

the string scale downto the TeV scale to avoid a hierarchy with the weak scale [2, 3]. The hierarchy problem is recast in geometric terms, namely the stabilization of the compactification size in very large volumes. These are difficult to detect since they are only felt by gravitational interactions. Present bounds on the size of 'gravity-only' extra dimensions come from tabletop experiments (like the Cavendish experiment), and impose only that their length scale in not larger than 0.1 millimeter. Notice however that a low string scale is not compulsory in models with some solution to the hierarchy problem, e.g. supersymmetric models.

2 Model building: Non-perturbative heterotic vacua

In this and the following section, we describe the basic rules for the construction of vacua of string theory or M-theory, with localized gauge sectors with features similar to those of the Standard Model. Explicit models with spectrum extremely close to that of the Standard Model have been constructed. However, in this lecture we will be happy by simply describing the apperance of charged chiral fermions, and the underlying reason for family replication. More detailed model building issues are left for the references.

We start by considering the setup provided by compactifications of Horava-Witten theory. This can be considered as the strong coupling limit of compactifications of the $E_8 \times E_8$ heterotic string theory, and hence most of the tools are already familiar. There are however some interesting new ingredients.

Consider M-theory compactified to 4d on $\mathbf{X_6} \times \mathbf{S^1/Z_2}$. In general we will be interested in supersymmetric models, hence we choose $\mathbf{X_6}$ to be a

Calabi-Yau threefold [3].

As in compactifications of the heterotic string theory, the

compactification is required to satisfy certain consistency conditions, arising from the equation of motion for some p-form fields. Namely, in heterotic theory the interactions for the NSNS 6-form B_6

$$\int_{10d} B_6 \wedge *B_6 + \int_{10d} B_6 \wedge (\mathrm{tr}\, F^2 - \mathrm{tr}\, R^2) \tag{9}$$

led to the equation of motion for the NSNS 2-form

$$dH_3 = \mathrm{tr}\, F^2 - \mathrm{tr}\, R^2 \tag{10}$$

In Horava-Witten theory, we need to consider two gauge bundles on the 10d boundaries of the interval, each with structure group a subgroup of E_8. The action for the 6-form C_6 (which is just the lift of the heterotic B_6) reads

$$\begin{aligned} S_{C_6} &= \int_{11d} *G_7 \wedge G_7 + \\ &+ \int_{11d} \delta(x^{10})(\mathrm{tr}\, F_{E_8}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 + \int_{11d} \delta(x^{10} - \pi R)(\mathrm{tr}\, F_{E_8'}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 = \\ &= \int_{11d} dG_4 \wedge C_6 + \\ &+ \int_{11d} \delta(x^{10})(\mathrm{tr}\, F_{E_8}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 + \int_{11d} \delta(x^{10} - \pi R)(\mathrm{tr}\, F_{E_8'}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 \end{aligned}$$

where $\delta(x)$ is a bump 1-form localized in the interval. We have a similar equation of motion for the M-theory 3-form, namely

$$dG_4 = \delta(x^{10})(\mathrm{tr}\, F_{E_8}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \; + \; \delta(x^{10} - \pi R)(\mathrm{tr}\, F_{E_8'}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \tag{11}$$

[3] A motivation for supersymmetry in this setup is that there is only one ‘gravity-only’ dimension. If we build a non-supersymmetric model, and try to lower the 11d Planck scale to the TeV range to avoid a hierarchy problem, we should take this dimension very large to generate a large 4d Planck scale. In fact, so large that it would conflict with the experimental bounds. Hence, a large 11d Planck scale is convenient in this setup, and supersymmetry is the most reasonable way to stabilize the weak scale against it. It however may be somewhat lower than the 4d Planck scale.

Taking this relation in cohomology, we obtain

$$[\mathrm{tr}\, F_{E_8}^2] + [\mathrm{tr}\, F_{E_8'}^2] - [\mathrm{tr}\, R^2] = 0 \quad \text{namely} \quad c_2(E) = c_2(R) \tag{12}$$

We would like to point out that the class of models is in fact richer. We can consider compactifications to 4d, where the background configuration also includes sets of k_a M5-branes [4] sitting at a point x_a^{10}in the interval, and with two of their world-volume dimensions wrapped on a 2-cycle $\Pi_a \subset \mathbf{X_6}$. Since the M5-branes are magnetically charged under the M-theory 3-form, the action for the 11d dual 6-form C_6 is

$$\begin{aligned} S_{C_6} &= \int_{11d} *G_7 \wedge G_7 + \sum_a k_a \int_{M_4 \times \Pi_a} C_6 + \\ &+ \int_{11d} \delta(x^{10})(\mathrm{tr}\, F_{E_8}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 + \int_{11d} \delta(x^{10} - \pi R)(\mathrm{tr}\, F_{E_8'}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 = \\ &= \int_{11d} dG_4 \wedge C_6 + \sum_a k_a \int_{11d} \delta(x^{10} - x_a^{10})\delta(\Pi_a) \wedge C_6 + \\ &+ \int_{11d} \delta(x^{10})(\mathrm{tr}\, F_{E_8}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 + \int_{11d} \delta(x^{10} - \pi R)(\mathrm{tr}\, F_{E_8'}^2 - \frac{1}{2}\mathrm{tr}\, R^2) \wedge C_6 \end{aligned}$$

where $\delta(\Pi_a)$ is a bump 4-form with support on the 2-cycle Π_a. The equation of motion for C_6, taken in cohomology gives the consistency condition for this kind of compactification, which reads

$$[\mathrm{tr}\, F_{E_8}^2] + [\mathrm{tr}\, F_{E_8'}^2] + \sum_a k_a[\delta(\Pi_a)] - [\mathrm{tr}\, R^2] = 0 \tag{13}$$

where $[\Pi_a]$ is the 4-cohomology class dual to the 2-homology class of the 2-cycle $[\Pi_a]$. Namely, M5-branes contribute to the condition of cancellation of 6-form charge, via the homology class of the 2-cycle they wrap.

[4]Notice that taking the limit of small interval size shows that this possibility is also available in heterotic theory. Hence, there exist compactifications of heterotic on Calabi-Yau threefolds, with NS5-branes. Due to the presence of the latter, these vacua are non-perturbative, even if the string coupling is small.

Compactifications with M5-branes have been studied in [4]. Since the M5-brane classes help in satisfying the consistency condition, it follows that there is additional freedom in the gauge bundles, and hence in the low-energy spectra of the theory. They lead to additional phenomena, for instance there may be transitions where some M5-brane moves towards the boundary in the interval and is diluted as an instanton class in the boundary gauge field. We will not go into these discussions.

Once the topology of the gauge bundles over the boundaries, namely their structure groups H, H', and characteristic classes, and the M5-brane configuration, are specified, the computation of the 4d massless spectrum is similar to that in heterotic theory.

• We obtain the 4d $\mathcal{N}=1$ supergravity multiplet, the dilaton chiral multiplet, and $(h_{1,1})+h_{2,1}$ chiral mulitplets arising from geometric moduli.

• We obtain vector multiplets for the gauge group given by the commutant of H, H' in E_8. Notice that the choice $H=SU(3)$, $H'=1$ still leads to $E_6\times E_8$, but does not correspond to embedding the spin connection into the gauge degrees of freedom, since the latter would involve both E_8 factos in a symmetric way.

• Charged chiral multiplets arise from the KK reduction of the 10d gaugino, and their multiplicity is given by the index of the Dirac operator coupled to the gauge bundle (in a representation corresponding to the their 4d gauge representation).

• There may be additional multiplets arising from the KK reduction of the M5-brane world-volume theory on the 2-cycle Π_a. These can be trickier to discuss, so we skip their details.

Taken overall, many of the features of these models are similar to compactifications of heterotic string theory. However, the existence of the 'gravity only' dimension allows to lower the fundamental scale somewhat below the

4d Planck scale.

3 Model building: D-brane-worlds

Another class of models with localized gauge sectors can be obtained by considering compactifications with D-branes. An additional advantage of these setups is that, for simple enough D-brane configurations (i.e. in the absence of curvatures) the quantization of open string sectors can be carried out exactly (in the sense of the α expansion).

A first issue that we should address is how to obtain D-brane sectors containing chiral fermions in the corresponding open string spectrum. In fact, the simplest D-brane configurations, like D-branes in flat space (or in toroidal compactifications), with trivial world-volume gauge bundle (zero field strength for world-volume gauge fields, preserve too much supersymmetry to allow for chirality (that is, they have at least 4d $\mathcal{N} = 2$ supersymmetry) [5].

In fact, we can heuristically argue that *isolated*D-branes sitting at a *smooth* point in transverse space lead to non-chiral open string spectra. Considering for instance the case of D3-branes, sitting at a smooth point P in Transverse 6d space $\mathbf{X_6}$, see figure 3. Since chiral matter is necessarily massless, if present it should arise from open strings located at P and stretching between the D3-branes. Hence, only the local behaviour of $\mathbf{X_6}$ around P is important. If P is smooth this local behaviour is that of $\mathbf{R^6}$, hence the massless open string sector if simply that on D3-branes in flat space, which

[5] One way to generate chiral fermions is in fact to consider introducing a non-trivial bundle for the D-brane world-volume gauge field, with support on the internal cycle Π_{p-3}wrapped by the Dp-brane. This kind of model is, in some respects (like in the computation of the spectrum, etc) similar to heterotic models, and we do not discuss it here.

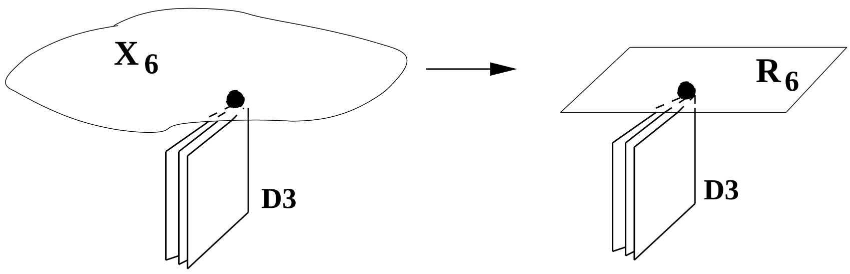

Figure 3: Isolated D-branes at a smooth point in transverse space feel a locally trivial geometry and lead to non-chiral open string spectra.

is non-chiral.

There are two ways which have been used in the construction of D-brane Configurations with chiral open string sectors; they arise from relaxing each of the above conditions in italic writing:

• Relaxing the smoothness condition, we may consider D-branes sitting at singular points in transverse space. The prototypical example is provided by a stack of D3-branes located at an orbifold singularity, $\mathbf{C^3/Z_N}$. See figure 4.

• Relaxing the condition of isolatedness, we may consider configurations of D-branes intersecting over subspaces of their world-volume. The prototypical case is provided by D6-branes intersecting over 4d subpaces of their world-volumes. See figure 5

In the following we discuss the appearance of chiral fermions, and the spectrum in these two kinds of D-brane configurations.

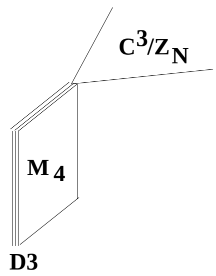

Figure 4: Stack of D3-branes at an orbifold singularity

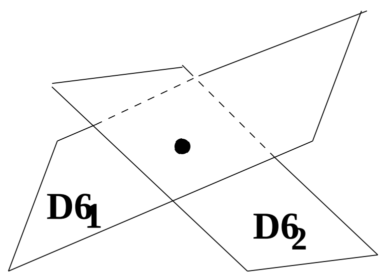

Figure 5: Two intersecting D6-branes in flat space.

3.1 D-branes at singularities

For concreteness, let us center of a stack of n D3-branes sitting at the Origin of a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity. These models were first Considered in [5]. The $\mathbf{Z_N}$ generator θ acts on the three complex coordinates of $\mathbf{C^3}$ as follows

$$(z_1, z_2, z_3) \to (e^{2\pi I\, a_1/N} z_1, e^{2\pi i\, a_2/N} z_2, e^{2\pi i\, a_3/N} z_3) \tag{14}$$

where the $a_i \in \mathbf{Z}$ in order to have and order N action [6]. We will center on orbifolds that preserve some supersymmetry, hence their holonomy must be in $SU(3)$ and thus we require $a_1 \pm a_2 \pm a_3 = 0$ mod N, for some choice of signs.

The closed string spectrum in the configuration can be obtained using the techniques explained in the corresponding lecture. Moreover, this sector will be uncharged under the gauge group on the D-brane world-volume, so it is not too interest for our discussion and we skip it.

Concerning the open string sector, the main observation is that there are no twisted sectors. This follows because the definition of twisted sectors in closed strings made use of the periodicity in the worldsheet direction σ, and this is not allowed in open strings. Hence, the spectrum of open strings on a set of D3-branes at a $\mathbf{C^3}/\mathbf{Z_N}$ orbifold singularity is simply obtained by considering the open string spectrum on D3-branes in flat space $\mathbf{C^3}$, and keeping the $\mathbf{Z_N}$-invariant ones. Each open string state on D3-branes in flat space is given by a set of oscillators acting on the vacuum, and an $n \times n$ Chan-Paton matrix λ encoding the $U(n)$ gauge degrees of freedom. The action of θ on one such open string state is determined by the action on the corresponding set of oscillators and the action on the Chan-Paton matrix. For concreteness,let us center on massless states. The eigenvalues of the

[6] One also needs $N \sum_i a_i$ = even (so that the quotient is a spin manifold, i.e. allows spinors to be defined).

different sets of oscillators for these states are

Sector	State	θ eigenvalue
NS	$(0,0,0,\pm)$	1
	$(\underline{+,0,0,}0)$	$e^{2\pi i\, a_i/N}$
	$(\underline{-,0,0,}0)$	$e^{-2\pi i\, a_i/N}$
R	$\pm\frac{1}{2}(+,+,+,-)$	1
	$\frac{1}{2}(\underline{-,+,+,}+)$	$e^{2\pi i\, a_i/N}$
	$\frac{1}{2}(\underline{+,-,-,}-)$	$e^{-2\pi i\, a_i/N}$

The eigenvalues can be described as $e^{2\pi i\, r\cdot v}$, where r is The $SO(8)$ weight and $v = (a_1, a_2, a_3, 0)/N$. The above action can easily be understood by decomponsing the $SO(8)$ representation with respect to the $SU(3)$ subgroup in which the $\mathbf{Z_N}$ is embedded. In fact we have $8_V = 3 + \overline{3} + 1 + 1$, and $8_C = 3 + \overline{3} + 1 + 1$, and noticing that (14) defines the action on the representation 3. Notice that the fact that bosons and fermions have the same eigenvalues reflects the fact that the orbifold preserves $\mathcal{N} = 1$ supersymmetry on the D-brane world-volume theory. In fact we see that the different states group into a vector multiplet V, with eigenvalue 1, and three chiral multiplets, Φ_i with eigenvalue $e^{2\pi i\, a_i/N}$.

On the other hand, the action of θ on the Chan-Paton degrees of freedom corresponds to a $U(n)$ gauge transformation. This is defined by a unitary order N matrix $\gamma_{\theta,3}$, which without loss of generality we can diagonalize and write in the general form

$$\gamma_{\theta,3} = \mathrm{diag}\,(1_{n_0}, e^{2\pi i/N} 1_{n_1}, \ldots, e^{2\pi i(N-1)/N} 1_{n_{N-1}}) \tag{15}$$

with $\sum_{a=0}^{N-1} n_a = n$. The action on the Chan-Paton wavefunction (which transforms in the adjoint representation) is

$$\lambda \to \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{16}$$

We now have to keep states invariant under the combined action of θ on the oscillator and Chan-Paton piece. For states in the $\mathcal{N}=1$ vector multiplet, the action on the oscillators is trivial, hence the surviving states correspond to Chan-Paton matrices satisfying the condition

$$\lambda = \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{17}$$

The surviving states correspond to a block diagonal matrix. The gauge group is easily seen to be

$$U(n_0) \times \ldots \times U(n_{N-1}) \tag{18}$$

For the i^{th} chiral multiplet Φ_i, the oscillator part picks up a factor of $e^{2\pi i\, a_i/N}$. So surviving states have Chan-Paton wavefunction must satisfy

$$\lambda = e^{2\pi i\, a_i/N} \gamma_{\theta,3} \lambda \gamma_{\theta,3}^{-1} \tag{19}$$

The surviving multiplets correspond to matrices with entries in a diagonal shifted by a_i blocks. It is easy to see that the surviving multiplets transform in the representation

$$\sum_{i=1}^{3} \sum_{a=0}^{N-1} (\square_a, \overline{\square}_{a+a_i}) \tag{20}$$

We clearly see that in general the spectrum is chiral, so we have achieved the construction of D-brane configurations with non-abelian gauge symmetries and charged chiral fermions. Moreover, we see that in general the different fermions have different quantum numbers. The only way to obtain a replication of the fermion spectrum (i.e. a structure of families, like in the Standard Model), we need some of the a_i to be equal (modulo N). The most interesting example is obtained for the $\mathbf{C^3}/\mathbf{Z_3}$ singularity, with $v=(1,1,-2)/3$. The spectrum on the D3-brane world-volume is given by

$$\begin{array}{rl} \mathcal{N}=1 \text{ Vect.Mult.} & U(n_0) \times U(n_1) \times U(n_2) \\ \mathcal{N}=1 \text{ Ch.Mult.} & 3\,[\,(n_0,\overline{n}_1,1) + (1,n_1,\overline{n}_2) + (\overline{n}_0,1,n_2)] \end{array} \tag{21}$$

we see there is a triplication of the chiral fermion spectrum. Hence in this setup the number of families is given by the number of complex planes with equal eigenvalue.

We would like to point out that, as usual in models with open strings, there exist some consistency conditions, known as cancellation of RR tadpoles. Namely, there exist disk diagrams, see figure 6, which lead to the coupling of D-branes at singularities to RR fields in the θ^k twisted sector. When the θ^k twist has the origin as the only fixed point, the corresponding RR fields do not propagate over any dimension transverse to the D-brane. This implies that they have compact support, and Gauss law will impose the corresponding charges must vanish, namely that the corresponding disk diagrams cancel. The coefficient of the disk diagram is easy to obtain: from the figure, we see that any worldsheet degree of freedom must suffer the action of θ^k as it goes around the closed string insertion. In particular it means that the Chan-Paton degrees of freedom suffer the action of $\gamma_{\theta^k,3=(\gamma_{\theta,3}}^k$ as they go around the boundary. Hence the disk amplitude is proportional to $\mathrm{tr}\,\gamma_{\theta^k,3}$, and the RR tadpole condition reads

$$\mathrm{Tr}\,\gamma_{\theta^k,3} = 0 \quad , \text{for } ka_i \neq 0 \,\mathrm{mod} N \tag{22}$$

For instance, for the above $\mathbf{Z_3}$ model these constraint require $n_0 = n_1 = n_2$. In general, the above constrains ensure that the 4d chiral gauge field theory on the volume of the D3-branes is free of anomalies.

Clearly the above model is not realistic. However, more involved models of this kind, with additional branes (like D7-branes, also passing through the singularity), can lead to models much closer to the Standard Model, see [6]. Their study is however beyond the topic of this lecture.

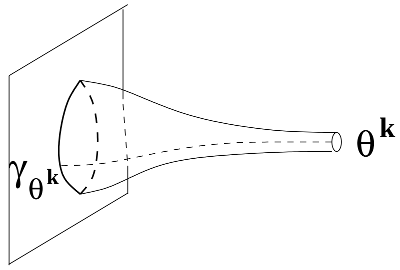

Figure 6: D3-branes at singularities are charged under RR forms in the θ^k twisted sector, via a disk diagram. Worldsheet degrees of freedom suffer the action of θ^k as they go around the cut, shown as a dashed line. The amplitude is proportional to $\mathrm{tr}\,\gamma_{\theta^k}$.

3.2 Intersecting D-branes

In this section we consider a different class of D-brane configurations leading to chiral 4d fermions. Consider two stacks of D6-branes (denoted $D6_1$- and $D6_2$-branes) in flat 10 space, intersecting over a 4d subspace of their world-volumes, see figure 7a. A slightly more explicit picture of the configuration is shown in figure 7b. The local geometry is determined by the three angles $\pi\theta_i$ that relate the two D6-branes in the 6d space transverse to the 4d intersection. For the following analysis, see [7].

Two such sets of D6-branes, intersecting at general angles, break all the supersymmetries of the theory. The supersymmetries preserved by one of the stacks are broken by the other, and vice versa. Consider the $D6_1$-branes to span the direction 0123456. The supersymmetry transformations unbroken by these D6-branes are of the form $\epsilon_L Q_L + \epsilon_R Q_R$ with

$$\epsilon_L = \Gamma^0 \ldots \Gamma^6 \epsilon_R \tag{23}$$

where the subindices L, R denote the supersymmetries arising from the left

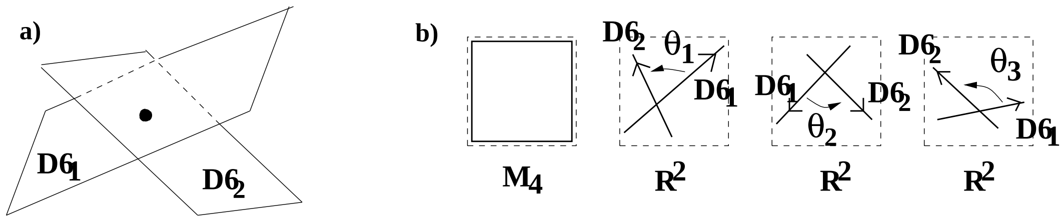

Figure 7: Two picture of D6-branes intersecting over a 4d subspace of their volumes.

or right movers. Denoting by R the $SO(6)$ rotation rotating the D6_1-branes to the D6_2-branes, the supersymmetries unbroken by the latter are

$$\epsilon_L = R^{-1}\,\Gamma^0 \ldots \Gamma^6\, R\,\epsilon_R \tag{24}$$

where here R denotes the action of the rotation in the spinor representation.

In general, there are no spinors surviving both conditions. However, for Special choices of the angles θ_i, i.e. of the rotation R, there may exist solutions to the above two conditions. In fact, it is easy to realize that if R is a rotation in an $SU(3)$ subgroup of $SO(6)$, there is one component of the spinor which is invariant under R, and both condition become identical. Therefore, intersections of D6-branes related by angles θ_i satisfying

$$\theta_1 \pm \theta_2 \pm \theta_3 = 0 \tag{25}$$

for some choice of signs, preserve 4 supercharges (1/4 of the supersymmetries preserved by the first stack of branes). This is the equivalent of 4d $\mathcal{N} = 1$, hence we may expect these configurations to lead to chiral 4d fermions. We will check below that this is indeed the case. Notice also that if the rotation is in a subgroup of $SU(2)$ (e.g. $\theta_1 \pm \theta_2 = 0$, $\theta_3 = 0$), the system preserves more spinors, in fact 8 supersymmetries, the equivalent of 4d $\mathcal{N} = 2$ supersymmetry.

Let us compute the spectrum of open strings in the above configuration of two Intersecting stacks of D6-branes, at generic angles θ_i. Consider open strings stretching among the N_1 D6_1-branes. This sector does not notice the presence of the second stack, so gives the same answers as for isolated D6-branes. We obtain $U(N_1)$ gauge bosons and their superpartners with respect to the 16 unbroken supersymmetries, propagating over the 7d volume of these D6-branes. For the sector of open strings stretching among the N_2 D6_2-branes, we similarly obtain $U(N_2)$ gauge bosons and their partners (under the 16 susys unbroken by the second D6-branes; notice these are not the same susy as above), propagating over the 7d volume of these D6-branes.

The novelty arises in the sector of open strings stretching between D6_1- and D6_2-branes. This sector feels both branes, and hence notices the amount of supersymmetry preserved by the two-stack system. We thus expect the spectrum in this sector to be non-supersymmetric for generic angles θ_i, and to gather into supermultiplets only for a constrained set of angles. Let us carry out the quantization of the sector of $6_1 6_2$ open strings. The only difference with respect to other open string sectors is in the boundary conditions. Consider two coordinates X_1, X_2 in a two-plane in which the D6-branes are rotated by an angle θ. The boundary conditions for the corresponding worldsheet fields for an open string are

$$\begin{aligned} \partial_\sigma X_1|_{\sigma=0} &= 0 \\ \partial_t X_2|_{\sigma=0} &= 0 \\ \cos\pi\theta\,\partial_\sigma X_1 + \sin\pi\theta\,\partial_\sigma X_2|_{\sigma=\ell} &= 0 \\ -\sin\pi\theta\,\partial_t X_1 + \cos\pi\theta\,\partial_t X_2|_{\sigma=\ell} &= 0 \end{aligned} \tag{26}$$

In complex coordinates $Z = X_i + iX_2$, we have

$$\partial_\sigma(\mathrm{Re}Z)|_{\sigma=0} = 0$$

$$\partial_t(\mathrm{Im}Z)|_{\sigma=0} = 0$$
$$\partial_\sigma(\mathrm{Re}e^{i\theta}Z)|_{\sigma=\ell} = 0$$
$$\partial_t(\mathrm{Im}e^{i\theta}Z)|_{\sigma=\ell} = 0 \qquad (27)$$

Imposing these boundary conditions on the open string oscillator expansion leads to the constraints that: the center of mass position of the open string is located at the intersection point; momentum and winding are necessarily zero; oscillators have moddings shifted by $\pm\theta$. Applying this rule to the three complex coordinates corresponding to intersecting D6-branes, we obtain oscillators $\alpha^i_{n+\theta_i}$, $\alpha^{\bar{i}}_{n-\theta_i}$ for the complexified 2d bosons, and $\Psi^i_{n+\nu+\theta_i}$, $\Psi^{\bar{i}}_{n+\nu-\theta_i}$ for the 2d fermions, with $n \in \mathbf{Z}$ and $\nu = 1/2, 0$ for the NS and R sectors. The computation of the spectrum is formally similar to the computation of the spectrum on the left movers in an orbifold. In particular the fractional modding of oscillators introduces a modified vacuum energy. The final result for the spectrum, centering on light states, is as follows (we assume $\theta_i \in (-1/2, 1/2)$)

Sector	State	$\alpha' M^2$	4d Lorentz
NS	$\Psi^1_{-1/2+\theta_1}\|0\rangle$	$\frac{1}{2}(-\theta_1+\theta_2+\theta_3)$	Scalar
	$\Psi^2_{-1/2+\theta_2}\|0\rangle$	$\frac{1}{2}(\theta_1-\theta_2+\theta_3)$	Scalar
	$\Psi^3_{-1/2+\theta_3}\|0\rangle$	$\frac{1}{2}(\theta_1+\theta_2-\theta_3)$	Scalar
	$\Psi^1_{-1/2+\theta_1}\Psi^2_{-1/2+\theta_2}\Psi^3_{-1/2+\theta_3}\|0\rangle$	$\frac{1}{\ }12(\theta_1+\theta_2+\theta_3)$	Scalar
R $\|0\rangle_R$	Weyl spinor		

All these fields propagate on the 4d intersection of the two D6-branes, and transform in the bifundamental representation (N_1, N_2) of the gauge group $U(N_1) \times U(N_2)$. The $6_2 6_1$ open string sector is quantized analogously, and in fact provides the antiparticles for the above fields. We see that generically bosons and fermions are unpaired, and only when the angles define a rotation in $SU(3)$ one of the bosons becomes massless and pairs up with the

4d fermion in the R sector, to give a 4d chiral multiplet. Notice that in the non-supersymmetric case, the scalars in the NS sector may have positive or negative mass square. If all scalars have positive mass square, the configuration of intersecting branes is stable. On the other hand, the existence of some tachyonic scalar signals an instability against a process in which the intersecting D6-branes recombine into a single smooth one. We will not say much more about this interesting process.

The important point in the above construction is that it provides a new setup with D-branes containing non-abelian gauge symmetries and charged chiral fermions. We now briefly describe how to exploit it in the construction of 4d models. For a review, see [8].

Although intersecting D6-branes provide 4d chiral fermions already in flat 10d space, gauge interactions remain 7d and gravity interactions remain 10d unless we consider compactification of spacetime. Hence, the general kind of configurations we are to consider (see figure 8) is type IIA string theory on a spacetime of the form $M_4 \times \mathbf{X_6}$ with compact $\mathbf{X_6}$, and with stacks of N_a D6_a-branes with volumes of the form $M_4 \times \Pi_a$, with $\Pi_a \subset \mathbf{X_6}$ a 3-cycle. It is important to realize that generically 3-cycles in a 6d compact space intersect at points, so the corresponding wrapped D6-branes will intersect at M_4 subspaces of their volumes. Hence, compactification reduces the 10d and 7d gravitational and gauge interactions to 4d, and intersections lead to charged 4d chiral fermions. Also, generically two 3-cycles in a 6d space intersect several times, therefore leading to a replicated sector of opens strings at intersections. This is a natural mechanism to explain/reproduce the appearance of replicated families of chiral fermions in Nature!

Denoting the 3-homology classes of the wrapped 3-cycles by $[\Pi_a]$, the intersection number is computed $I_{ab} = [\Pi_a] \cdot [\Pi_b]$, computed as described in the lecture on topology. The 4d spectrum on the resulting configuration is

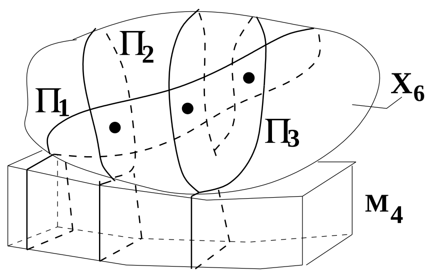

Figure 8: Compactification with intersecting D6-branes wrapped on 3-cycles.

easy to obtain. From the sector of open strings strecthing among the $D6_a$-branes, we obtain the KK reduction on Π_a of the 7d $U(N_a)$ gauge bosons and partners. In general we obtain 4d $U(N_a)$ gauge bosons [7]. From the sector of open string stretching between the a^{th} and b^{th} stacks of D6-branes, we obtain a chiral 4d fermion in the bifundamental for each intersection of the corresponding 3-cycles. There are in general additional light scalars, which may become massless if the intersection is locally supersymmetric (ie the intersection angles define a rotation in $SU(3)$). Taken overall, the (chiral part of the) 4d spectrum is

$$\begin{array}{cl} \text{Gauge} & \prod_a U(N_a) \\ \text{Left.Ch.Fm.} & \sum_{a<b} I_{ab}(\Box_a, \overline{\Box}_b) \end{array} \tag{28}$$

We note that a negative intersection number indicates the fermions have the opposite chirality.

These models have to satisfy some consistency conditions, namely cancellation of RR tadpoles. The D6-branes act as sources for the RR 7-forms

[7]Plus some partners if the 3-cycle Π_a is special lagrangian, i.e. the wrapped D-brane preserves some supersymmetry. We will not enter into this discussion.

via the disk coupling $\int_{W_7} C_7$. The consistency condition amounts to requiring the total RR charge of D-branes to vanish, as implied by Gauss law in a compact space (since RR field fluxlines cannot escape). The condition of RR tadpole cancellation can be expressed as the requirement of consistency of the equations of motion for RR fields. In our situation, the terms of the spacetime action depending on the RR 7-form C_7 are

$$\begin{aligned} S_{C7} &= \int_{M_4\times \mathbf{X_6}} H_8 \wedge *H_8 \,+\, \sum_a N_a \int_{M_4\times \Pi_a} C_7 \,= \\ &= \int_{M_4\times \mathbf{X_6}} C_7 \wedge dH_2 \,+\, \sum_a N_a \int_{M_4\times \mathbf{X_6}} C_7 \wedge \delta(\Pi_a) \end{aligned} \tag{29}$$

where H_8 is the 8-form field strength, H_2 its Hodge dual, and $\delta(\Pi_a)$ is a bump 3-form localized on Π_a in $\mathbf{X_6}$. The equations of motion read

$$dH_2 \,=\, \sum_a N_a\, \delta(\Pi_a) \tag{30}$$

The integrability condition is obtained by taking this equation in homology, yielding

$$[\Pi_{\rm tot}] \,=\, \sum_a N_a\, [\Pi_a] \,=\, 0 \tag{31}$$

As usual, cancellation of RR tadpoles in the underlying string theory configuration implies cancellation of four-dimensional chiral anomalies in the effective field theory in our configurations.

Let us provide one simple example, obtained by taking $\mathbf{X_6} = \mathbf{T^6}$, and a simple set of 3-cycles. We consider $\mathbf{X_6}$ to be a six-torus factorized as $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$. Also for simplicity we take the 3-cycles Π_a to be given by a factorized product of 1-cycles in each of the 2-tori. For a 3-cycle Π_a, the 1-cycle in the i^{th} 2-torus will be labeled by the numbers (n_a^i, m_a^i) it wraps along the horizontal and vertical directions, see figure 9 for examples.

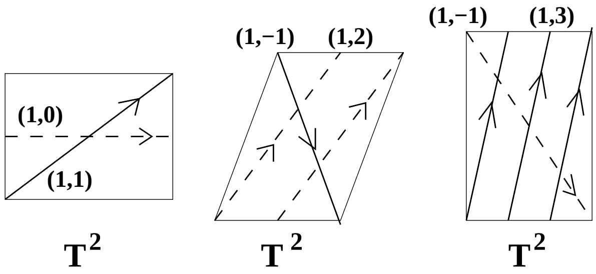

Figure 9: Examples of intersecting 3-cycles in $\mathbf{T^6}$.

The intersection number is given by the product of the number of intersections in each 2-torus, and reads

$$I_{ab} \;=\; (n_a^1 m_b^1 - m_a^1 n_b^1) \times (n_a^2 m_b^2 - m_a^2 n_b^2) \times (n_a^3 m_b^3 - m_a^3 n_b^3) \tag{32}$$

To give one interesting example, consider a configuration of D6-branes on $\mathbf{T^6}$ defined by the following wrapping numbers

$$\begin{array}{cccc}
N_1 = 3 & (1,2) & (1,-1) & (1,\text{-}2) \\
N_2 = 2 & (1,1) & (1,-2) & (\text{-}1,5) \\
N_3 = 1 & (1,1) & (1,0) & (\text{-}1,5) \\
N_4 = 1 & (1,2) & (-1,1) & (1,1) \\
N_5 = 1 & (1,2) & (-1,1) & (2,\text{-}7) \\
N_6 = 1 & (1,1) & (3,-4) & (1,\text{-}5)
\end{array}$$

The intersection numbers are

$$\begin{array}{ccccc}
I_{12} = 3 & I_{13} = -3 & I_{14} = 0 & I_{15} = 0 & I_{16} = -3 \\
I_{23} = 0 & I_{24} = 6 & I_{25} = 3 & I_{26} = 0 & I_{34} = -6 \\
I_{35} = -3 & I_{36} = 0 & I_{45} = 0 & I_{46} = 6 & I_{56} = 3
\end{array}$$

A $U(1)$ linear combination, playing the role of hypercharge, remains massless

$$Q_Y = -\frac{1}{3} Q_1 - \frac{1}{2} Q_2 - Q_3 - Q_5 \tag{33}$$

The chiral fermion spectrum, with charges with respect to the Standard Model - like gauge group, is

$$\begin{gathered} SU(3) \times SU(2) \times U(1)_Y \times \ldots \\ 3(3,2)_{1/6} + 3(\overline{3},1)_{-2/3} + 3(\overline{3},1)_{1/3} + 6(1,2)_{-1/2} + \\ +3(1,2)_{1/2} + 6(1,1)_1 + 3(1,1)_{-1} + 9(1,1)_0 \end{gathered} \tag{34}$$

Notice however, that the model contains additional $U(1)$ factors and other gauge factors, as well as matter beyond the context of the Standard Model.

In any event this general setup therefore allows the construction of a large class of models with 4d gravitational and non-abelian gauge anomalies, and charged chiral fermions. We leave their more detailed discussion for the interested reader (see [8] for a review) and simply point out that, although most models constructed in this setup are non-supersymmetric, there exist several explicit supersymmetric examples in the literature.

4 Final comments

The main message of this lecture is that there exist constructions in string and M-theory which have the potential of leading to low-energy physics very close to that observed in Nature. Perturbative heterotic string are simply one such setup, but there are others, like compactifications of Horava-Witten theory, or models with D-branes. There is life beyond perturbative heterotic theory!

The novelty about these new setups is that they have localized gauge sectors, and hence allow for fundamental scales not directly tied up to the 4d Planck scale, and can even be significantly lower then the latter. In models with a too low fundamental scale, there may be dangerous processes, like too fast proton decay. In many of the D-brane models above, there exist some symmetries which forbit this violation of baryon number.

The models are also interesting in that they provide an essentially new way to obtain gauge symmetries and chiral fermions in string theory. In particular this can be exploited to imagine new sources for the hierarchy of Yukawa couplings and fermion masses in the standard model.

Besides these novelties and successes, it is however important not to loose perspective and recognize that the models still leave many unanswered questions.

- If supersymmetry is present, how to break supersymmetry? If not, how to stabilize moduli at values that may correspond to (seemingly unnatural) large volumes?

- The moduli problem: Or how to get rid of the large number of massless scalars which exist in many compactifications in string theory (and whose vevs encode the parameters of the underlying geometry and gauge bundle (like sizes of the internal manifold, etc)).

- The vaccum degeneracy problem: Or the enormous amount of consistent vacua which can be constructed, out of which only one (if any at all) is realized in the real world. Is this model preferred by some energetic, cosmological, anthropic criterion? Or is it all just a matter of chance?

- The cosmological constant problem, which in general is too large once we break supersymmetry. Does string theory say anything new about this old problem?

As one can notice, the list is 'isomorphic' to the one we had in perturbative heterotic models. This means that certainly these are difficult problems which permeate any model buiding setup in string theory. Clearly we need better theoretical understanding of new aspects theory. This is not impossible, however, as for instance there are recent proposals to stabilize most compactification moduli by studying compactifications with non-trivial field strength fluxes for p-form fields [10]. Thus the above problems, which are central questions in string phenomenology, will hopefully solved perhaps by next-generation students like you!

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