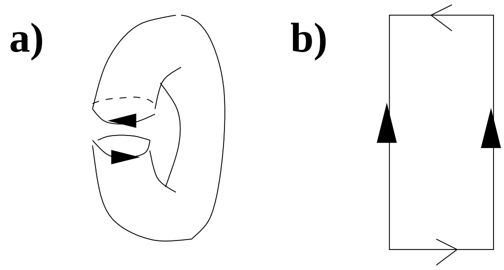

Figure 1: Two pictures representing the Klein bottle. In b) we construct it as a rectangle with vertical sides identified with the same orientation and horizontal sides glued with the reversed orientation, as suggested by the arrow.

Type I superstring

1 Unoriented closed strings

1.1 Generalities

Consider a closed oriented string theory which is left-right symmetric, e.g. closed bosonic string theory or type IIB theory. Consider modding it out, quotienting, by the operation Ω, worldsheet parity, that exchanges left and right movers. Namely, construct the quotient theory, where states related by left-right exchange are considered equivalent

$$|a\rangle_L \otimes |b\rangle_R \quad |a\rangle_R \otimes |b\rangle_R \tag{1}$$

This operation is called orientifolding the theory by Ω (this is also called gauging the global symmetry Ω).

The genus expansion in the quotient theory is drastically different from the original one. Consider for instance 1-loop vacuum diagrams. As usual we

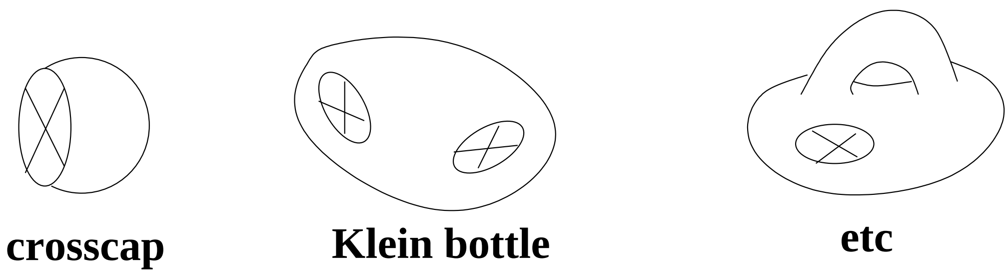

Figure 2: Several examples of non-orientable surfaces constructed by glueing crosscaps to a sphere.

have the torus, which corresponds to closed string states $A_L \times B_R$ which evolve and are glued back to the original state. In theories where states related by Ω are considered equivalent, there is a new diagram. It corresponds to starting with a closed string state $A_L \times B_R$ letting it evolve and glueing it back to the original up to the action of Ω. This is shown graphically in figure 1 where we can see the worldsheet is a non-orientable surface, a Klein bottle. The result generalizes to other amplitudes as the statement that the genus expansion of unoriented theories contains non-orientable worldsheets.

A general worldsheet (including oriented and unoriented ones) can be described as a sphere with an arbitrary number of handles and crosscaps. A crosscap can be described as cutting a small disk in the surface and identifying antipodal points in the resulting boundary to close back the surface. Several non-orientable surfaces are shown in figure 2. In theories with open string sectors (see later) the genus expansion contains worldsheets with boundaries. Recalling the discussion in the review lectures, an amplitude mediated by a worldsheet with g handles, n_c crosscaps and n_b boundaries is weighted by a factor of $e^{-\xi\phi}$, where ϕ is the dilaton vev and $\xi = 2 - 2g - n_c - n_b$ is the Euler characteristic of the worldsheet.

The spectrum of the unoriented theory is obtained from the spectrum of the 'parent' oriented theory very simply. Namely, one takes the original spectrum and keeps only the Ω invariant states (or linear combinations of states). The same result is obtained from our description of the genus expansion: One way to obtain the spectrum of a theory is to see what states contribute in the one-loop vacuum amplitude. The sum over the two contributions, the torus and the Klein bottle, can be written in terms of the original Hilbert space of states as

$$\begin{aligned} & \mathrm{tr}_{\mathcal{H}_{\mathrm{oriented}}} (\ldots) + \mathrm{tr}_{\mathcal{H}_{\mathrm{oriented}}} (\ldots\,\Omega), = \\ & = \mathrm{tr}_{\mathcal{H}_{oriented}} [\ldots\, \tfrac{1}{2}(1+\Omega)] \end{aligned} \qquad (2)$$

the piece $\frac{1}{2}(1+\Omega)$ is a projector that only keeps Ω invariant states, so that only the later contribute to the amplitude. For non-invariant states, the contributions from the torus and Moebius strip cancel each other; the sum over topologies projects out those states.

1.2 Unoriented closed bosonic string

Let us obtain the precise action of Ω on closed string states in a systematic way (to be used in other cases as well). The action of Ω on the 2d bosonic field $X(\sigma,t)$ is to transform it into a field $X^{i\prime} = \Omega X^i \Omega^{-1}$ such that

$$X^{i\prime}(\sigma,t) \;=\; X^i(\ell-\sigma,t) \qquad (3)$$

Introducing the oscillator expansion

$$X^i(\sigma,t) \;=\; x^i + \frac{p^i}{p^+}t + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left[\frac{\alpha_n^i}{n} e^{-2\pi i\, n(\sigma+t)\ell} + \frac{\tilde{\alpha}_n^i}{n} e^{2\pi i\, n(\sigma-t)\ell} + \right] \qquad (4)$$

we obtain

$$x^{i\prime} = x^i \quad ; \quad p^{i\prime} = p^i \quad ; \quad \alpha_n^{i\,\prime} = \tilde{\alpha}_n^i \quad ; \quad \tilde{\alpha}_n^{i\,\prime} = \alpha_n^i \qquad (5)$$

which corresponds to an exchange of the left and right movers, as expected.

The quotient theory is obtained by taking the vacuum of the original theory

$$\alpha_n^i|0\rangle = 0 \quad ; \quad \tilde{\alpha}_n^i|0\rangle = 0 \quad \forall n > 0 \tag{6}$$

and applying left and right oscillators forming Ω invariant states. The space-time mass of these states is given by the original formula

$$\alpha' m^2/2 \;=\; N_B \;+\; \tilde{N}_B \;-\; 2 \tag{7}$$

The lightest modes are

State	$\alpha' m^2/2$	Lorentz rep
$\|0\rangle$	-2	scalar
$\alpha_{-1}^{(i}\tilde{\alpha}_{-1}^{j)}\|0\rangle$	0	graviton
$\sum_i \alpha_{-1}^i\tilde{\alpha}_{-1}^i\|0\rangle$	0	dilaton

We see that the 2-form of the original theory is odd under Ω and is projected out. The complete spectrum is easily obtained.

This concludes the construction of our theory, which can be checked to be completely consistent. In the following sections we try to construct an unoriented version of the (type IIB) superstring.

1.3 Unoriented closed superstring theory IIB/Ω

The worldsheet theory is in this case described by the 2d bosonic and fermionic fields $X^i(\sigma,t)$, $\psi^i(\sigma,t)$. The bosonic fields are discussed exactly as above. On the fermionic fields, the action of Ω is such that

$$\psi^{i\prime}(\sigma,t) \;=\; \psi^i(\ell-\sigma,t) \tag{8}$$

Using the oscillator expansion

$$\psi^i(\sigma,t) \;=\; i\sqrt{\frac{\alpha}{2}}\sum_{r\in\mathbf{Z}}\;\left[\psi^i_{r+\nu}\,e^{-2\pi i\,(r+\nu)(\sigma+t)/\ell}\;+\;\tilde{\psi}^i_{r+\nu}\,e^{2\pi i\,(r+\nu)(\sigma-t)/\ell}\right] \tag{9}$$

where $\nu=1/2,0$ for NS and R fermions, we obtain

$$\psi^i_{r+\nu}{}' = \tilde{\psi}^i_{r+\nu} \quad ; \quad \tilde{\psi}^i_{r+\nu}{}' = \psi^i_{r+\nu} \quad ; \tag{10}$$

We can now obtain the spectrum of the unoriented theory, which is simply obtained by taking the Ω invariant states of the original theory. There is an interesting subtlety in the action of Ω on RR states; since the left and right moving pieces in this sector are spacetime spinors, they anticommute, so that a state $A_L\times B_R$ is mapped by Ω to $A_R\times B_L=-B_L\times A_R$. The Ω invariant states are therefore of the form $A_L\times B_R-B_L\times A_R$. Notice also that states in the NS-R sector must combine with states in the R-NS sector to form invariant combinations.

The light spectrum is given by

Sector	State	$SO(8)$	Field
NS-NS	$\psi^{(i}_{-1/2}\|0\rangle\,\otimes\psi^{j)}_{-1/2}\|0\rangle$	$1+35_v$	dilaton, graviton
NS-R+R-NS	$\psi^i_{-1/2}\|0\rangle\otimes\tilde{8}_C\;+\;8_C\otimes\tilde{\psi}^i_{-1/2}\|0\rangle$	56_S+8_S	gravitinos
R-R	$[8_C\otimes\tilde{8}_C]$	28_C	2-form

This spectrum corresponds to the gravity multiplet of 10d $\mathcal{N}=1$ supergravity. Notice in particular that the orientifold projection kills one linear combination of the two gravitinos of the original $\mathcal{N}=2$ supersymmetric type IIB theory.

This theory as it stands is clearly not consistent. A theory whose spectrum is just the gravity multiplet of 10d $\mathcal{N}=1$ theory has 10d gravitational

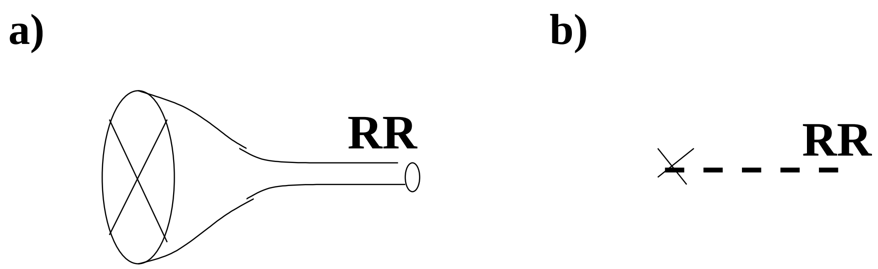

Figure 3: Crosscap diagram leading to a tadpole term for some closed string mode.

chiral anomalies. Clearly we have missed an important consistency condition in the construction of the theory.

The consistency condition is RR tadpole cancellation. Unoriented theories contain a diagram, given by a crosscap with an infinite tube attached to it (see fig. 3), which leads to a tadpole for certain massless fields. In particular there is a tadpole for a RR field, which due to 10d Poincare invariance, must be the non-propagating 10-form C_{10} (which can be seen to survive the orientifold projection). This RR tadpole renders the theory inconsistent.

The fact that the problem in constructing a theory of just unoriented closed strings is very similar to the problem of constructing a theory of open strings coupled to type IIB theory leads to the following suggestion. One can attempt to construct a theory free of RR tadpoles by considering the Ω orientifold of type IIB coupled to a sector of open strings. Namely, we can attempt to construct a theory where the RR tadpoles for C_{10} arising from open string sectors (disk diagrams) and unorientability (crosscap diagrams) cancel each other. This is the so-called type I superstring theory.

In other words, the equation of motion from the action for the 10-form

$$S_{C_{10}} = (Q_{\rm crosscap} + Q_{rmdisks}) \int C_{10} \tag{11}$$

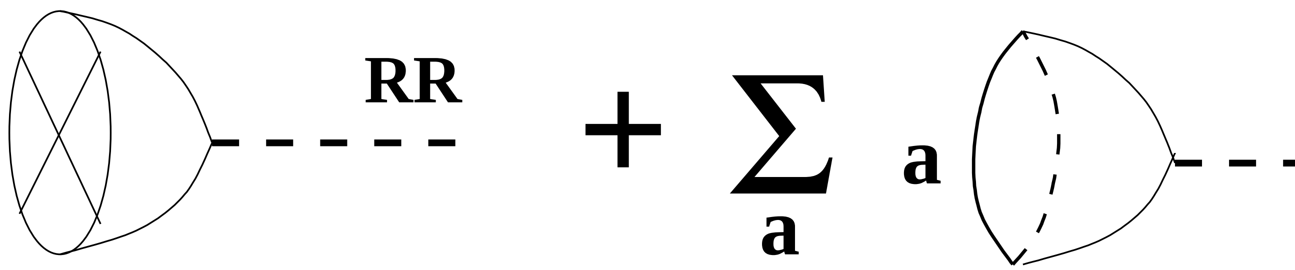

Figure 4: Cancellation of RR tadpoles from crosscap and disk diagrams.

would be satisfied

$$Q_{\text{crosscap}} + Q_{\text{disk}} = 0 \tag{12}$$

This is pictorially shown in figure 4. Open string sectors coupling to unoriented closed string must be unoriented as well. Hence if one is able to construct such a theory, it would be a theory of unoriented open and closed strings. Hence we need to know a bit about unoriented open strings before the final construction.

2 Unoriented open strings

2.1 Action of Ω on open string sectors

As mentioned in previous lectures, the local structure on the 2d worldsheet for open strings should be the same as for the corresponding closed sector. Hence, the action on the bosonic coordinates is such that

$$X^{i\prime}(\sigma, t) = X^i(\ell - \sigma, t) \tag{13}$$

Using the oscillator expansion for open strings,

$$X^i(\sigma,t) \;=\; x^i \;+\; \frac{p^i}{p^+}t \;+\; i\sqrt{\frac{\alpha'}{2}}\sum_n \frac{\alpha_n^i}{n}\cos\frac{\pi n\sigma}{\ell}\, e^{-\pi i n t/\ell} \tag{14}$$

we obtain

$$x^{i\prime} = x^i \quad ; \quad p^{i\prime} = p^i \quad ; \quad \alpha_n^{i\,\prime} = (-1)^n \alpha_n^i \tag{15}$$

The action on fermions is such that

$$\psi^{i\prime}(\sigma,t) = \psi^{'}(\ell-\sigma,t) \tag{16}$$

Using the expansion

$$\psi^i(\sigma,t) \;=\; i\sqrt{\frac{\alpha'}{2}}\sum_{r\in\mathbf{Z}}\left[\psi_{r+\nu}e^{-\pi i(r+\nu)(\sigma+t)/\ell} \;+\; (-1)^{2\nu}\,\psi_{r+\nu}e^{\pi i(r+\nu)(\sigma-t)/\ell}\right] \tag{17}$$

with $\nu = 1/2, 0$ for NS and R fermions, resp, we obtain

$$\psi^{i\prime}{}_{r+\nu} = (-1)^{r+\nu}\psi_{r+\nu} \tag{18}$$

It should be pointed out at this stage that there is a non-trivial action of Ω on the open string NS groundstate, namely

$$\Omega|0\rangle_{NS} = e^{-i\pi/2}|0\rangle_{NS} \tag{19}$$

Finally, we also need to specify the action of Ω on the Chan-Paton indices in cases where they are present. Clearly Ω exchanges the order of the labels ab, since it reverses the orientation of the open string.

A general state with fixed operator structure may be written as a linear combination of the corresponding state in the different open string sectors, of the form $\lambda_{ab}|ab\rangle$. The $N\times N$ matrix λ_{ab} is known as the Chan-Paton wavefunction of the state. The action of Ω on Chan-Paton labels can be encoded into an action on λ

$$\lambda \xrightarrow{\Omega} \gamma_\Omega \lambda^T \gamma_\Omega^{-1} \tag{20}$$

where γ_Ω is an $N \times N$ unitary matrix or order two. There are two canonical choices, distinguished by the symmetry of γ_Ω

$$\begin{aligned} \text{i)} \quad \gamma_\Omega &= \mathbf{1}_N \\ \text{ii)} \quad \gamma_\Omega &= \begin{pmatrix} 0 & i\mathbf{1}_{N/2} \\ -i\mathbf{1}_{N/2} & 0 \end{pmatrix} \end{aligned} \tag{21}$$

The first option i) is also often described as

$$\gamma_\Omega = \begin{pmatrix} 0 & \mathbf{1}_{(N/2} \\ \mathbf{1}_{N/2} & 0 \end{pmatrix} \text{ for } N = \text{even} \; ; \; \gamma_\Omega = \begin{pmatrix} 1 & & \\ & 0 & \mathbf{1}_{(N-1)/2} \\ & \mathbf{1}_{(N-1)/2} & 0 \end{pmatrix} \text{ for } N = \text{odd} \tag{22}$$

A more transparent interpretation of these actions on Chan-Paton labels is as follows (we take N even for simplicity). Consider splitting the set of labels into two sets, running from 0 to $N/2$ and from $N/2+1$ to N, and label them by indices a, and a'. Denoting the Chan Paton index part of a state by e.g. $|ab\rangle$, the actions above are

$$\begin{aligned} |ab\rangle \to |b'a'\rangle \quad &; \quad |a'b'\rangle \to |ba\rangle \\ |ab'\rangle \to \pm|ba'\rangle \quad &; \quad |a'b\rangle \to \pm|ba'\rangle \end{aligned} \tag{23}$$

with $+, -$ signs for symmetric or antisymmetric γ_Ω.

2.2 Spectrum

It is now easy to obtain the spectrum of the unoriented open string sector, by simply keeping the states of the original theory invariant under the combined action of Ω on the oscillator operators, the vacuum and the Chan Paton labels. We center on the massless sector.

In the NS sector, the states $\lambda\psi^i_{-1/2}|0\rangle$ transform as

$$\lambda\, \psi^i_{-1/2}\, |0\rangle \;\xrightarrow{\Omega}\; -\,\gamma_\Omega \lambda^T \gamma_\Omega^{-1}\, \psi^i_{-1/2}\, |0\rangle \tag{24}$$

Invariant states correspond to components of the matrix λ surviving the projection

$$\lambda = -\gamma_\Omega \lambda^T \gamma_\Omega^{-1} \tag{25}$$

In case i), we obtain $\lambda = -\lambda^T$, so there are $N(N-1)/2$ surviving gauge bosons. This number, and the relation with antisymmetric matrices as generators, suggest that the gauge bosons fill out a gauge group $SO(N)$.

In case ii), writing $\lambda = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$, the projection imposes $A = -D^T$, $B = B^T$, $C = C^T$. There are $N(N+1)/2$ gauge bosons, and this suggests that the gauge group is $USp(N)$.

In the R sector, the GSO projection selects the groundstate transforming as 8_C. The action of Ω turns out to introduce a minus sign on it, so the projection condition on λ is again

$$\lambda = -\gamma_\Omega \lambda^T \gamma_\Omega^{-1} \tag{26}$$

Hence in cases i) and ii) we get 10d fermions in the adjoint representation of $SO(N)$ and $USp(N)$ respectively. The NS and R sectors altogether give an $SO(N)$ or $USp(N)$ vector multiplet of 10d $\mathcal{N}=1$ supersymmetry. So the open string sector preserves the same amount of supersymmetry as the unoriented closed string sector.

3 Type I superstring

As discussed above, the idea in the construction of type I superstring is to add (unoriented) open string sectors to the unoriented closed string theory in section 1, in such a way that the contribution of disks and crosscaps to the 10-form RR tadpole cancels.

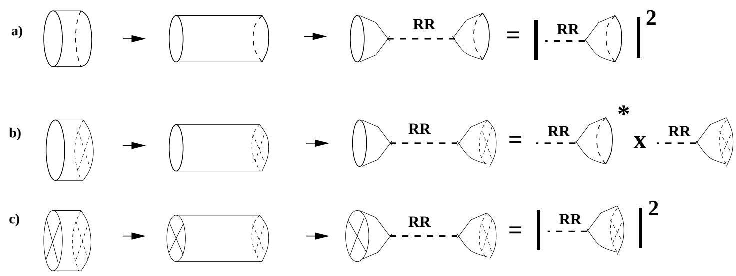

Figure 5: Disk and crosscap tadpoles can be recovered in the factorization limit of certain one-loop amplitudes, namely the annulus (a), the Moebius strip (b) and the Klein bottle (c).

3.1 Computation of RR tadpoles

The idea

Instead of computing directly the disk and crosscap diagrams with insertions of the massless RR field, there is an indirect but standard way of computing them. In particular it is useful in making sure the disk and crosscaps come out with the same normalization (which is clearly crucial to have correct cancellation).

The idea is that since we are interested in computing e.g. the disk with insertion of a massless field, this can be recovered from an annulus amplitude with no insertions, in the limit in which it factorizes in the closed string channel. this is shown in figure 5a). Similarly, the amplitude for a crosscap with insertion of massless fields can be recovered from the factorization limit of diagrams in figure 5b,c. These diagrams, as we discuss later on, correspond to a Moebius strip and a Klein bottle.

Indeed computing a sum of these diagrams of closed strings propagating for some time $T'\ell$ between disks and crosscaps, as shown in figure 6a), and taking the factorization limit $T' \to \infty$ one recovers the expression for the

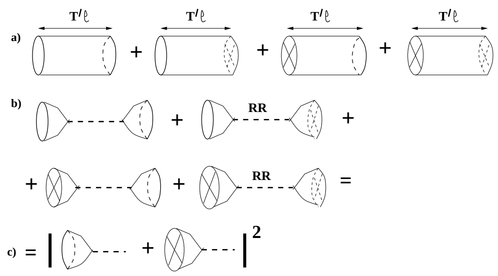

Figure 6: The sum of four amplitudes factorizes as the square of the total disk plus crosscap tadpole.

square of the total RR tadpole. This is pictorially shown in fig 6, and holds very precisely in the explicit computation to be discussed later on.

These diagrams are most easily computed in the dual channel, where they reduce to traces over Hilbert spaces. The channel in figure 6 is recovered by performing a modular transformation, after which we may take the factorization limit. Let us consider the different surfaces

The annulus

The diagram with two disks is our old friend the annulus. It can be easily computed as an ampiltude for an open string to travel for some time $2T\ell$ and glue back to itself. Taking into account the trace over Chan-Paton indices, it reads

$$Z_A = N^2 \int_0^\infty \frac{dT}{2T} \operatorname{tr}_{\mathcal{H}_{\rm open}} e^{-2T\ell H_{\rm open}} \tag{27}$$

The trace is over open oriented string states (since it is the sum over worlsheets that implements the orientifold projection, we do not have to impose it explicitly). We have

$$\operatorname{tr}_{\rm mom.} e^{-2\pi\alpha' T \sum_i p_i^2} \quad = (8\pi^2\alpha' T)^{-4}$$

$$
\begin{aligned}
\mathrm{tr}_{\,\mathrm{bos.}}\, e^{-2\pi T(N_B - E_0^B)} &= \eta(iT)^{-8} \\
\mathrm{tr}_{\,NS,GSO}\, e^{-2\pi T(N_F - E_0^F)} &= \frac{1}{2}\left(\mathrm{tr}_{\,NS} q^{N_F + E_0^F} + \mathrm{tr}_{\,NS}(q^{N_F+E_0^F}(-)^F\right) = \\
&\frac{1}{2}\eta^{-4}\left(\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}^4 - \vartheta\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^4\right) \\
\mathrm{tr}_{\,R,GSO}\, e^{-2\pi T(N_F - E_0^F)} &= \frac{1}{2}\left(\mathrm{tr}_{\,R} q^{N_F + E_0^F} + \mathrm{tr}_{\,R}(q^{N_F+E_0^F}(-)^F\right) = \\
&\frac{1}{2}\eta^{-4}\left(\vartheta\begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^4 - \vartheta\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^4\right)
\end{aligned} \tag{28}
$$

In total

$$
Z(T) = \frac{1}{2}(8\pi^2\alpha'\tau_2)^{-4}\,\eta^{-8}\,\eta^{-4}\left(\vartheta\begin{bmatrix} 0 \\ 0 \end{bmatrix}^4 - \vartheta\begin{bmatrix} 0 \\ 1/2 \end{bmatrix}^4 - \vartheta\begin{bmatrix} 1/2 \\ 0 \end{bmatrix}^4 + \vartheta\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}^4\right) \tag{29}
$$

As shown in figure 7, in going to the dual channel we find a closed string propagating between two disks during a time $T'\ell$ with $T' = \frac{1}{2T}$. We should then replace $T = \frac{1}{2T'}$ in the above expression. To make the formula look like an amplitude in the dual channel we should perform a modular transformation. Leaving the details for a second version of these notes, the amplitude will read

$$
Z_A = \int_0^\infty \frac{dT'}{2T'}\,\tilde{Z}_A(2T') \tag{30}
$$

In this amplitude it is easy to identify the propagation of RR modes (upper characteristic of the theta function is 1/2). Taking the limit $T' \to \infty$ in this piece leads to

$$
Z_A \to N^2 \tag{31}
$$

This is proportional to the square of the RR disk tadpole.

Klein bottle

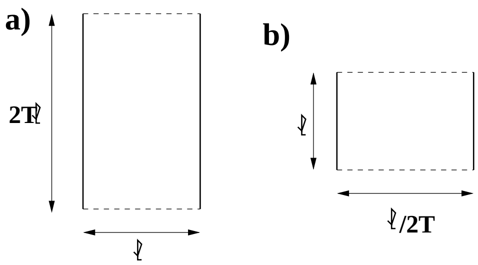

Figure 7: An open string propagating a time $2T\ell$ is geometrically the same as a closed string propagating a time $T'\ell$ with $T' = 1/(2T)$.

The Klein bottle amplitude corresponds to a closed string that propagates for a time $T\ell$ and is glued back to itself up to the action of Ω, see figure 1. The measure is obtained from that of the torus noticing that Ω does not allow for the τ_1 parameter. The amplitude hence reads

$$Z_K = \int_0^\infty \frac{dT}{4T} \, \mathrm{tr}_{\,\mathcal{H}_{\text{closed}}} \, e^{-T\ell H_{\text{closed}}} \tag{32}$$

The sum is over the Hilbert space of closed oriented strings. However, states non-invariant under Ω can be written as a sum over an Ω-even and an *Omega*-odd state

$$|A\rangle = \frac{1}{2}(|A\rangle + \Omega|A\rangle) + \frac{1}{2}(|A\rangle - \Omega|A\rangle) + \tag{33}$$

which have the same energy and different Ω eigenvalue. Hence their contributions cancel in the trace. Consequently, only states direcly mapped to themselves by Ω can contribute. Since this states are exactly left-right symmetric, we can simply sum over left-moving states and double the energy of each state. We obtain

$$\begin{aligned} Z_K(T) &= \mathrm{tr}_{\,\text{mom.}} e^{-\pi\alpha' T \sum_i p_i^2} \, \mathrm{tr}_{\,\text{bos.}} \, e^{-2\pi T(N_B - E_0^B)} \times \\ &\times \left(\mathrm{tr}_{\,NS,GSO} \, e^{-2\pi T(N_F - E_0^F)} - \mathrm{tr}_{\,R,GSO} \, e^{-2\pi T(N_F - E_0^F)} \right) \end{aligned} \tag{34}$$

The result is

$$Z(T) \;=\; \frac{1}{2}\,(4\pi^2\alpha' T)^{-4}\,\eta^{-8}(2iT)\;\eta^{-4}\left(\vartheta\begin{bmatrix}0\\0\end{bmatrix}^4 - \vartheta\begin{bmatrix}0\\1/2\end{bmatrix}^4 - \vartheta\begin{bmatrix}1/2\\0\end{bmatrix}^4 + \vartheta\begin{bmatrix}1/2\\1/2\end{bmatrix}^4\right)(2iT$$

The Klein bottle is topologically the same surface as a closed string propagating between two crosscaps. This is shown in fig 8. In this dual closed channel the closed string propagates for a time $T'\ell$ with $T' = \frac{1}{4T}$. Replacing T in the amplitude and perform a modular transformation (for details, see a forthcoming second version of these notes), the amplitude will read

$$Z_K \;=\; \int_0^\infty \frac{dT'}{2T'}\,\tilde{Z}_K(2T') \tag{36}$$

Extracting the contribution from RR modes and taking $T' \to \infty$ in leads to

$$Z_K \to (32)^2 \tag{37}$$

This is proportional to the square of the RR crosscap tadpole, with same proportionality as in (31).

Moebius strip

The Moebius strip corresponds to an aa open string propagating from a time $2T\ell$ and glueing back to itself up to the action of Ω. This kind of diagram does not exist for ab states with $a \neq b$. The amplitude reads

$$Z_M \;=\; \pm N\int_0^\infty \frac{dT}{2T}\,\mathrm{tr}\,_{\mathcal{H}_{\rm open}}\left(e^{-2T\ell H_{\rm open}}\,\Omega\right) \tag{38}$$

The sign is given by the action of Ω on aa states, it can also be written $\mathrm{tr}\,(\gamma_\Omega^{-1}\gamma_\Omega^T)$ and is $+,-$ for cases i), ii) above.

The trace is over open oriented string states. However, in analogy with the Klein bottle, only states directly invariant under Ω contribute to the trace.

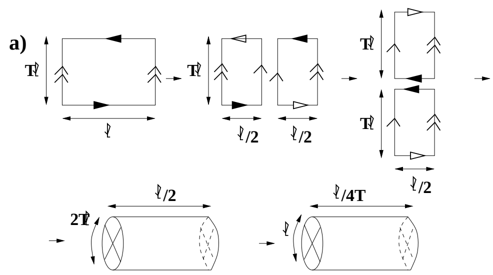

Figure 8: Take a Klein bottle as a rectangle with sides identified; cut it in two pieces keeping track of how they were glued; then glue explicitly some of the original identified sides. The result is the same surface now displayed as a surface with two crosscaps.

The explicit evaluation of this amplitude is easy, but involves slightly more complicated combinations of theta functions than the previous ones. We leave the details for a second version of these notes, and proceed the discussion in a qualitative way.

As shown in figure 9, the Moebius strip is topologically the same surface as a closed string propagating between a disk and a crosscaps. In this dual closed channel the closed string propagates for a time $T'\ell$ with $T' = \frac{1}{8T}$. Replacing T in the amplitude and perform a modular transformation, the amplitude will read

$$Z_M = \int_0^\infty \frac{dT'}{2T'} \, \tilde{Z}_M(2T') \tag{39}$$

Extracting the contribution from RR modes and taking $T' \to \infty$ in leads to

$$Z_K \to \mp 32 \, N \tag{40}$$

with $-,+$ corresponding to the cases i), ii) above. This is proportional to the product of the RR disk and crosscap tadpoles, with same proportionality as in (31).

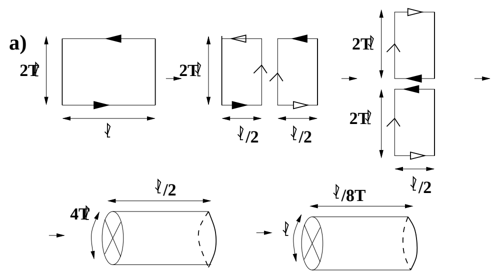

Figure 9: Take a Moebius strip as a rectangle with sides identified; cut it in two pieces keeping track of how they were glued; then glue explicitly some of the original identified sides. The result is the same surface now displayed as a surface with one boundary and one crosscap.

RR tadpole cancellation

The sum of the four amplitudes in fig 6a in the factorization limit is hence proportional to $(N-32)^2$. This implies that to obtain a consistent theory of unoriented open and closed strings, we need the Chan-Paton indices to run over 32 possible values

$$N = 32 \tag{41}$$

and the Ω action on them, γ_Ω, to be a symmetric matrix. This is type I superstring theory.

The spectrum of this theory is obtained straightforwardly. At the massless level the closed string sector corresponds to the 10d $\mathcal{N}=1$ supergravity multiplet, and the open string sector corresponds to 10d $\mathcal{N}=1$ vector multiplets with gauge group $SO(32)$.

Sector	Sector	$SO(8)$	Field
Closed	NS-NS	$1 + 35_V$	dilaton, graviton
	NS-R+R-NS	$8_S + 56_S$	gravitino
	R-R	28_C	2-form
Open	NS	8_V	$SO(32)$ gauge boson
	R	8_C	gauginos

Notice that this spectrum if free of gravitational and gauge anomalies. For this to be true, it is crucial that the gauge group is $SO(32)$, as we already saw in the discussion of anomalies in the heterotic theories. (interestingly enough, the massless spectrum of the $SO(32)$ and the type I string theories are the same).

In the cancellation of mixed gauge - gravitational anomalies, it is crucial the existence of a Green-Schwarz mechanism. Although at the level of the effective action the description for type I is similar to the one for heterotic (with the difference that the 2-form mediating the interaction is the RR one in type I theory), the string theory origin of the relevant couplings is different. In particular, both the BF^2 and BF^4 terms in type I string theory arise from disk diagrams with open string state insertions (powers of F) and a closed string B-field insertion, see figure 10.

4 Final comments

Just as with the other superstrings, there exist non-supersymmetric versions of type I superstring. One posibility is to construct orientifold quotients of the type 0 superstrings. We will not discuss these theories in our lectures. Another possibility [?] is to perform a modified Ω projection of type IIB theory which breaks the supersymmetries. We may discuss this theory later

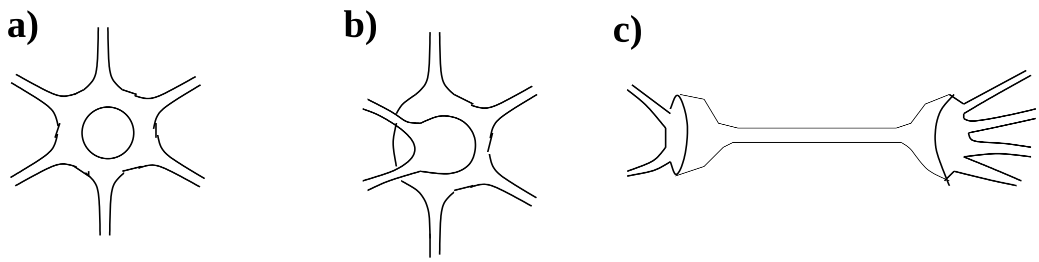

Figure 10: Limits of the annulus leading to anomalies in type I theory; a) corresponds to the familiar planar hexagon contribution to irreducible anomalies in field theory, while b) corresponds to a non-planar hexagon field theory contribution anomalies. c) corresponds to a Green-Schwarz diagram exchanging the closed string 2-form field, and which contributes to reducible anomalies.

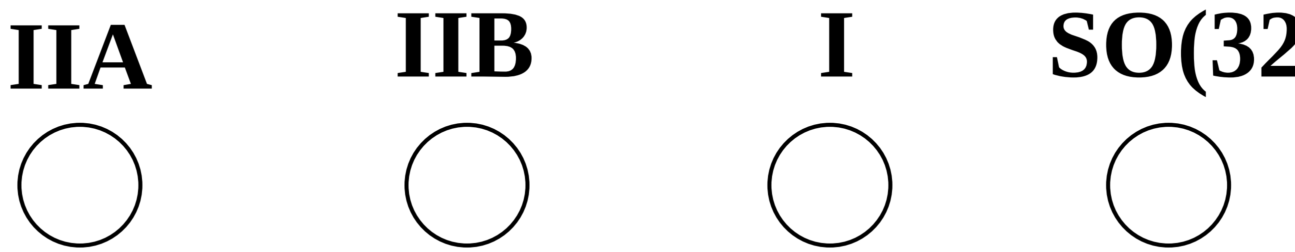

Figure 11: .

on in these lectures, since it will be easier to describe it once we learn about D-branes, orientifold planes and antibranes.

This concludes our discussion of the 10d superstring theories. At the moment the picture of string theory that we have is shown in fig 11. Five different (spacetime supersymmetric) superstring theories, constructed in different ways and with different features. All of them provide theories which describe gravitational (plus other) interactions in a quantum mechanically consistent way. However this multiplicity is unappealing: we would like to have a more unified description of how to construct consistent theories of gravitational interactions.

In the following lectures we will see that this picture will be drastically modified once we learn about compactification, T-duality and non-perturbative dualities. It turns out that the seemingly different string theories are intimately related, and seem to be just different limits of a unique underlying theory.

It would be very nice if the non supersymmetric strings would also fit into this unified picture. Although there are some ideas in the market, it is much more difficult to find evidence for this proposal.

References